

TANGRAMS

RONALD C. READ

What is a tangram? It is more easily seen and understood than defined in words. But to put it briefly, it is a geometric dissection puzzle in which seven standard pieces (five triangles, one square, one rhomboid) are used to make images of various objects. The origin of the name is not known, but the game itself is of Chinese origin, and for centuries has been one of the most popular diversions in the Orient.

The tangram is not entirely new to America, since Yankee sea captains brought books of tangram puzzles back from Canton and Shanghai. Edgar Allan Poe was a devotee, while on the other side of the world Napoleon is said to have whiled away his time with them. Around the turn of the present century the two great modern puzzlers, H. E. Dudeney and Sam Loyd (*Sam Loyd's Eighth Book of Tan*, Dover, \$1.00) greatly enlarged the traditional field of tangram situations.

The present collection by Ronald C. Read gathers together some 330 tangrams, the best creations of both Chinese and Occidental puzzle devisers. Included are some of the most striking Oriental puzzles, carefully selected from two 19th-century Chinese books, and some of the most fantastic, imaginative and whimsical inventions of Loyd and Dudeney. Although individual tangrams have always been favorite members of most collections of mathematical amusements, to our knowledge this is the first book devoted entirely to this popular form in the past 50 years.

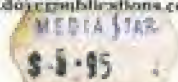
Some of the tangrams in this collection are relatively easy, and can be solved without too much brain-straining. Others are difficult, and may demand quite a bit of mental sweat before they are resolved. All, however, are delightful exercises in recreational form-perception, and will provide many hours of pleasure.

An original publication. 334 puzzles, all with solutions. v. + 152pp. 5 1/2 x 8 1/2. Paperbound.

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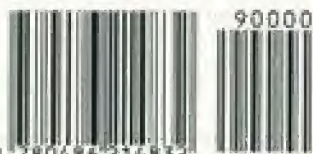
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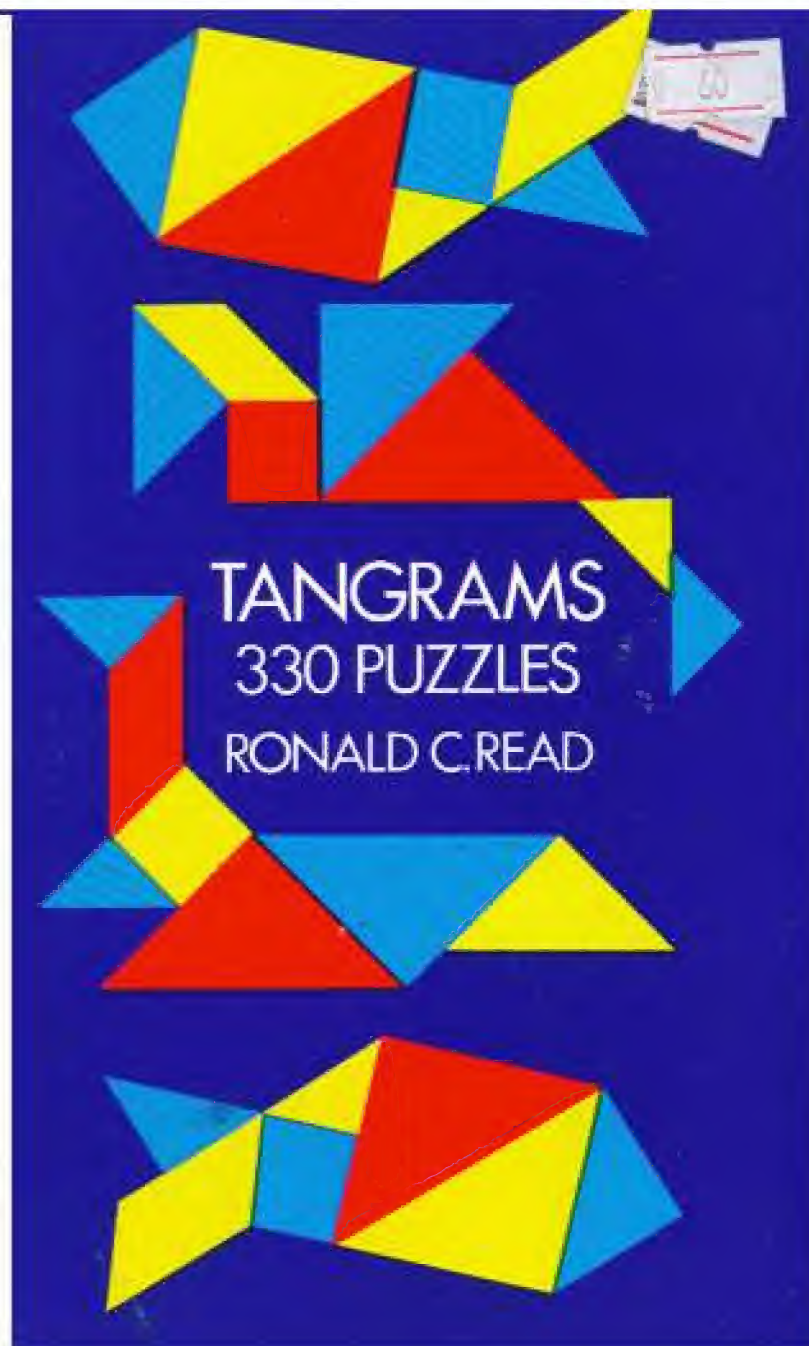


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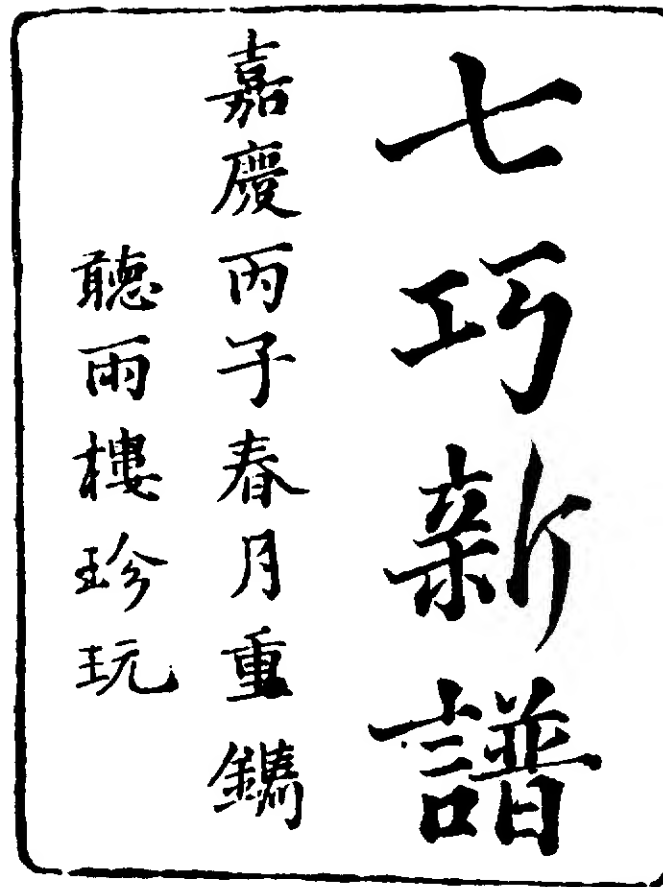
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TANGRAMS—

330 Puzzles

by Ronald C. Read



The title page of a Chinese tangram book (see page 59).

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Introduction

THE seven-piece puzzle, or tangram as it is usually known in the West, has come to us from China. It is in all probability the original “Chinese puzzle”—the proverbial prototype of all that is perplexing and tricky. This puzzle resembles the familiar jigsaw puzzle in that it is concerned with the fitting together of geometrical shapes, yet in all other respects the two puzzles are completely different; for whereas the jigsaw puzzle consists of a large number of pieces of rather complicated shapes, which have to be fitted together in a unique way, the tangram consists of only seven pieces of simple shapes, and the whole charm of the puzzle lies in the extraordinary variety of ways in which these pieces can be put together.

The seven pieces which make up the tangram can be cut from a single square, as shown in Figure 1. There are thus two small triangles, one medium-sized triangle, and two large triangles, in addition to a square and a lozenge-shaped piece.

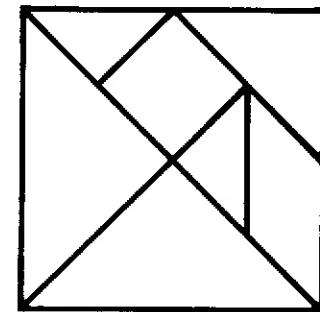


FIGURE 1

The medium-sized triangle and the square and the rhomboid are all twice the area of one of the small triangles; each of the

large triangles is four times the area of a small triangle. All the angles in these pieces are either right angles or angles of 45° or 135° . A set of tangram pieces can easily be made from cardboard. However, the enthusiast will probably want to make his set from wood, plastic or some other more durable material.

With a set of seven tangram pieces, the reader can make (if he tries hard enough) all the outlines that are to be found in the following pages except those of the last two chapters, and a quick glance through these pages will reveal the surprisingly large number of different outlines that can be constructed from such a small number of pieces. This is what has made the tangram popular in the country of its origin and also in the West. From time to time it has been revived on both sides of the Atlantic and has enjoyed a certain vogue, even though it has never achieved the status of such established pastimes as checkers or tic-tac-toe.

The tangram has a reputation for being very old, and this may well be, though one should beware the common tendency to ascribe exaggerated age to all things Chinese. I have in my possession two books of tangrams printed in China in 1813 and 1823. In the preface to one of these the author says that "the origin of the seven-piece puzzle is not known . . ." This would suggest that the puzzle was even then regarded as old, though just how old is a matter for conjecture. Despite this, I have been quite unable to find any Chinese books on tangrams, or even any reference to this kind of puzzle, dating earlier than the beginning of the nineteenth century, at which time a large number of books on the subject seem to have been published.

There was, apparently, a resurgence of interest in the tangram in China around this time, resulting in the publication in book form of what may previously have been handed down only by word of mouth. It should be mentioned here that the Chinese have always regarded the seven-piece puzzle as predominantly a game for children and women, and therefore perhaps hardly a worthy subject for books. I venture to hope that this is not true of the present book, which contains many

subtleties devised by occidental puzzlers that are not to be found in the Chinese texts.

It was during the first half of the nineteenth century that the tangram became known in Western countries, and books about it started to appear. At first, these books were nothing more than exact copies of Chinese texts, with the addition of hard covers and some sort of English preface explaining—often not very clearly—what the puzzle was about. A copy of one such book, now in the British Museum, bears on its cover the following:

A Grand Eastern Puzzle

THE following Chinese Puzzle is recommended to the Nobility, Gentry and others, being superior to any hitherto invented for the Amusement of the Juvenile World, to whom it will afford unceasing recreation and information: being formed on Geometrical principles, it may not be considered as trifling to those of mature years, exciting interest, because difficult and instructive, imperceptibly leading the mind on to invention and perseverance. — The Puzzle consists of five triangles, a square, and a rhomboid, which may be placed in upwards of THREE HUNDRED and THIRTY Characters, greatly resembling MEN, BEASTS, BIRDS, BOATS, BOTTLES, GLASSES, URNS, etc. The whole being the unwearied exertion of many years study and application of one of the Literati of China, and is now offered to the Public for their patronage and support.

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Later on, publications appeared in Europe and America which were more original, but which still depended very heavily on the earlier sources. The same outlines appear, often in the same order, and often with mistakes (common in the Chinese

books) faithfully copied! One such publication which I have in my possession was printed in Philadelphia in 1844. Its derivation from the earlier Chinese books is quite apparent, but it is greatly inferior to them. Approximately three hundred tangrams are given, but they are jumbled together any which way, and no indication is given of what the outlines are supposed to represent. This reduces the puzzle to the mere construction of meaningless geometrical patterns, which must have provided rather dull entertainment.

Lewis Carroll had in his library a book of tangrams entitled *The Fashionable Chinese Puzzle*, which seems to have been very similar to the American publication just mentioned. The great English puzzler H. E. Dudeney, who later acquired this book, describes it as follows:

It contains three hundred and twenty-three Tangram designs, mostly nondescript geometrical figures, to be constructed from the seven pieces. . . . There is no date, but the following note fixes the time of publication pretty closely: "This ingenious contrivance has for some time past been the favourite amusement of the ex-Emperor Napoleon, who, being now in a debilitated state and living very retired, passes many hours a day in thus exercising his patience and ingenuity."

This reference to Napoleon's interest in the puzzle occurs also in the preface to the Philadelphia publication, but it would seem unlikely that there is any truth in the assertion. Napoleon's biographers, writing of his exile on St. Helena, mention that he was interested (though not skilled) in billiards, chess and *reversi*; nothing is said of anything at all resembling the tangram. Still, it is not impossible that Napoleon was acquainted with this form of entertainment, and the story of the Emperor whiling away his declining years in this manner, if not true, is at all events a pleasing enough fabrication.

All this goes to show that historical information about the tangram is almost nonexistent; but though fact is in short supply there is fiction in abundance. A detailed and highly colored "tangram legend" has come into existence, mainly due to the

efforts of Sam Loyd, that great American puzzle expert of the turn of the century, aided and abetted by H. E. Dudeney in England. Sam Loyd's contribution to the literature of tangrams (and a very notable one it is, too) was a book called *The 8th Book of Tan*, which was published by Loyd himself in 1903. In this book Sam Loyd brought his own peculiar genius to bear on all aspects of the puzzle; in addition to creating hundreds of completely new shapes, he also dreamt up some paradoxical twists and quirks that had never been thought of before.

Along with these tangrams Sam Loyd supplied a running commentary, in which he gave what has every appearance of being a remarkably precise and carefully documented account of the history of the puzzle, its esoteric religious significance and symbolism, and its relation to the theorem of Pythagoras and other parts of Euclid's geometry, not to mention much additional scholarly and erudite information.

In point of fact, however, this painstaking work of scholarship is one delightful spoof from beginning to end! One example will suffice to show the sort of tall yarn that Sam Loyd pulls out of the hat. He quotes the researches of the "famous Professor Challenor" (a purely fictitious character, as far as I have been able to discover), who is said to have made a special study of the seven-piece puzzle. According to these researches the puzzle originated with a monumental work in seven volumes entitled *The Seven Books of Tan*, which was compiled in China some four thousand years ago. These books are described as "rare" (an understatement), and mention is made of one of them, printed in gold leaf upon parchment, that was found in Peking. Now, none of this is even remotely plausible. In 2000 B.C. the Chinese were still at the stage in which their literary achievements mainly consisted of crude pictographs scratched on tortoise shells; they had certainly not progressed even to writing on thin strips of bamboo—the way in which the Confucian classics, for example, were originally recorded (around 500 B.C.). A work of seven volumes would be quite out of the question at that time, and a book printed in gold

leaf on parchment would be most unusual (to say the least) at any time.

I must beg the reader's indulgence if he considers that this is all too obvious to be worth the trouble of explanation. I take the trouble because it would seem that H. E. Dudeney accepted these tall stories at their face value; if he did not, he at least repeated them with such seriousness and seeming credence that his readers, faced with the unanimity of two famous authorities, might well believe them to be true.

This, of course, is exactly what Sam Loyd would like. His spoofing is clearly deliberate, and is completely in keeping with the good-natured leg-pulling that lends special appeal to so many of his puzzles.*

Dudeney's long preamble to Problem 169 (A Tangram Paradox) in his *Amusements in Mathematics*† repeats the summary of Professor Challenor's researches and describes the attempts (by the eminent philologist Sir James Murray) to check on this story. Needless to say, the result was that the legendary "Tan" and his seven books are completely unknown in China. Dudeney then goes on, apparently in all innocence, to contribute an additional piece of spurious mystification to the "tangram legend." He refers to a Chinese book of tangrams about which an American correspondent has written to him, and he reproduces a Chinese inscription from its first page. His reason for doing this is that—

The owner of the book informs me that he has submitted it to a number of Chinamen in the United States and offered as much as a dollar for a translation. But they all steadfastly refused to read the words, offering the lame excuse that the inscription is Japanese. Natives of Japan, however, insist that it is Chinese. Is there something occult and esoteric about Tangrams, that it is so difficult to lift the veil?

* Two volumes of Loyd's puzzles have been republished recently, in which this propensity for fooling the reader is amply demonstrated. I refer to *Mathematical Puzzles of Sam Loyd* (two volumes), edited by Martin Gardner. New York: Dover Publications, Inc.

† Reprinted by Dover Publications Inc.



FIGURE 2

Now it so happens that I have a copy of this same book. The "mysterious inscription" is shown in Figure 2, along with the rest of the page on which it occurs. First of all we should remark that Dudeney, or his correspondent, has mistaken the back of the book for the front—a common error, since Chinese books start with what we would consider to be the last page. The inscription is actually on the last page but one. It is very badly printed, but otherwise there is nothing particularly difficult about it. One can only suppose that the people who were asked to translate it were less well versed in Chinese (or Japanese) than the owner of the book thought. The inscription reads roughly as follows: "Two men facing each other drinking. This shows the versatility of the seven-piece puzzle."

Clearly, this is simply a caption for the tangram outlines which appear beneath it. Far from being mysterious or occult, the inscription is not even out of the ordinary, since *every* outline

in the book has a caption! What a shame! Another lovely theory ruined by hard facts!

Despite the popular proverb, truth is seldom stranger than fiction, and the full history of the tangram puzzle is probably not nearly as colorful as the legend that Sam Loyd and Dudeney have woven around it. But this in no way detracts from the enjoyment that we can derive from this pastime, as the following pages are intended to show. The object of the game is, of course, to put the seven tangram pieces together so as to form the various outlines given throughout the book. Some of these are quite easy, others less so. If an outline really stumps you, look up the solution at the back of the book. But the Solutions should be your last resort; don't consult them unless you are really stuck. You may want to try to construct some original tangrams, and some interesting possibilities are suggested here and there in the text.

Finally, a word about the name "tangram" itself. The "gram" part presents no difficulty; it is a common ending denoting something written or drawn (as in "diagram," for example). The origin of the first part of the word is more in doubt. There are several explanations, but one is so much more plausible than any other as to be almost certainly correct. The Tang dynasty was one of the greatest in Chinese history; so much so that in certain South Chinese dialects the word *tang* (or *tong*) is synonymous with "Chinese." Now, a European visiting China in the first half of the nineteenth century, if he learned any Chinese at all, would be more likely to learn a Southern dialect than any other. Thus a traveler bringing the puzzle from China to the West, and wanting a name for it, would be very likely to take the word *tang*—that is, Chinese—and combine it with the familiar ending "gram." Dudeney quotes Sir James Murray as being of the opinion that the name was probably introduced in this way by an American some time between 1847 and 1864, the latter date marking the word's first appearance in Webster's Dictionary.

If this conjecture about the derivation of the name tangram is correct, then the tangram is truly the "Chinese puzzle," in name as well as in origin.

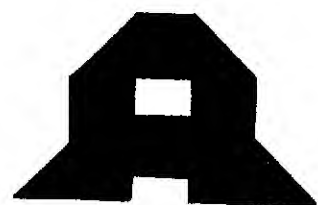
1. Letters and Numbers

THE seven tangram pieces can be fitted together in such a vast number of ways that in preparing a selection of them it is very difficult to know where to start. Probably as good a way as any is to go right back to our early school days, back to the days when we first learned the alphabet. This time we shall make the letters out of the tangram pieces instead of with pencil and paper.

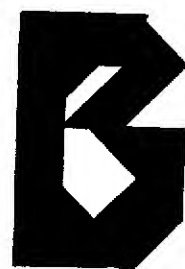
In the following pages you will find tangram outlines for all the letters of the alphabet. See if you can make them all yourself before looking at the solutions which begin on page 94.

Reconstructing tangram outlines that have been obtained by others is only half the fun that can be derived from this puzzle; as much entertainment, if not more, can be obtained by inventing new outlines. The reader can try his hand at this right away by trying to improve some of the outlines given for the letters. Certainly several could stand improvement; the N, Q and X, in particular, are nothing like as satisfactory as one would wish. The outlines given are the best that I have been able to concoct, but the reader may well be more successful.

For good measure, the digits 1, 2, up to 8 have also been given. Since zero is the same as the letter o, and 9 is simply a 6 upside down, these two digits are not new outlines, and have not been included.



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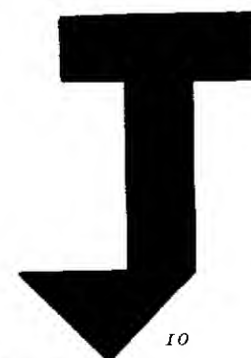
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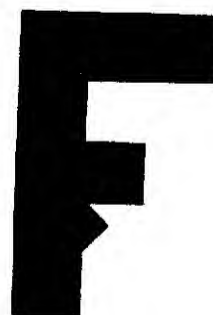
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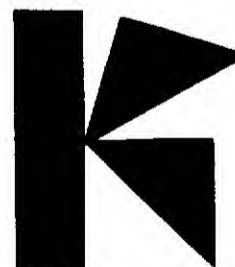
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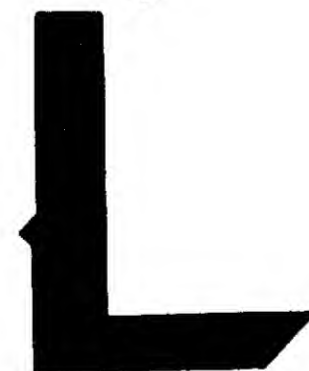
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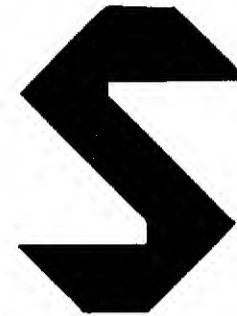
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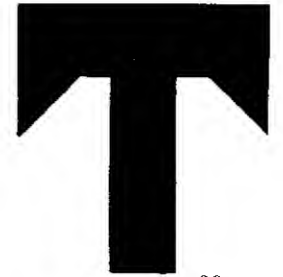
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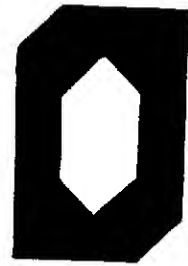


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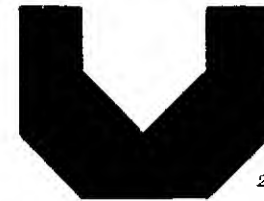
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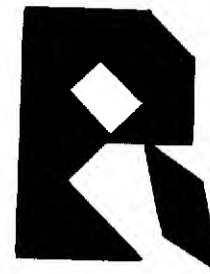
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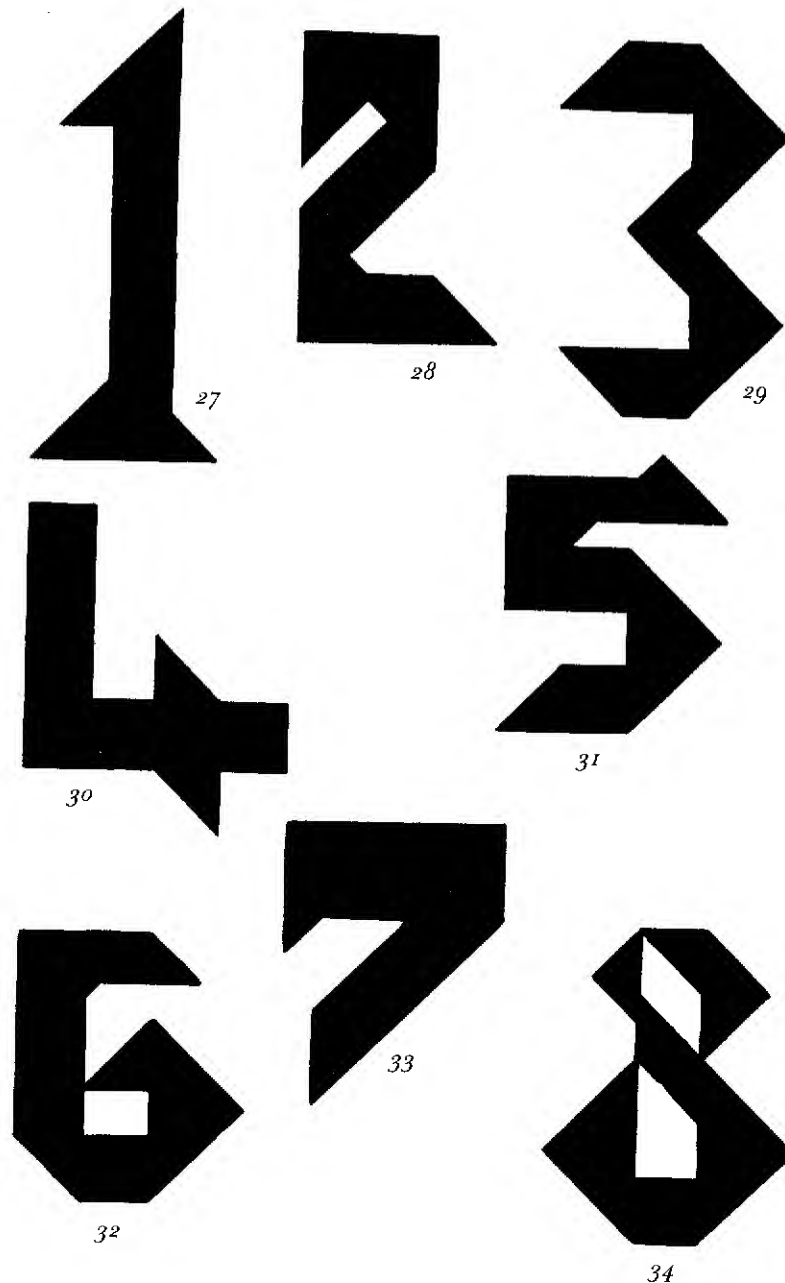
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25



26



2. *Animals*

THE Chinese tangram books, and the later Western books all contain large numbers of outlines representing animals. This book will be no exception, and our contribution to the tangram menagerie lies in the next few pages. Some of the outlines which follow are taken, or adapted, from the Chinese books, and some from Sam Loyd's book. As far as I know, the others are entirely new.

It might be thought that the tangram pieces, with their sharp angles and straight sides, could not possibly portray the complicated curved shapes of living creatures, but it is truly surprising how a well-designed tangram can suggest curves where none exist, and complexity where there is only simplicity. See, for instance, how a single piece suffices to suggest the back-curving horns of the mountain goat (No. 35); notice the stratagem by which the thin legs of the stork (No. 50) have been portrayed by three tangram pieces which, individually, are much thicker; note how the careful arrangement of the pieces unerringly conveys the fright of the startled cat (No. 41), the haughtiness of the camel (No. 47) and the graceful lines of the shark (No. 78). We shall see many examples throughout the book of outlines which, by their careful construction, convey much more to the mind's eye than is actually there.

In addition to the animals that have already been mentioned we have:

36. A horned animal of some kind. (Let's not inquire too closely into its precise genus and species!)

37. A polar bear.

38. A giraffe.

39 to 44. Assorted cats, closely followed (naturally) by 45 and 46. Two dogs.

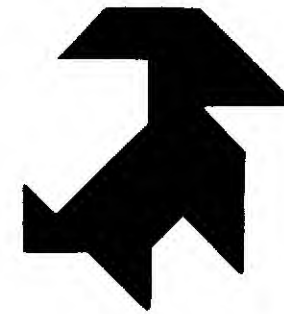
48. A squirrel.

- 49 to 61. Birds of various shapes and sizes.
 62 to 65. Four sinister-looking vultures.
 66 to 68. Three horses. Also No. 74.
 69 to 72. Four different kinds of bats.
 73. A kangaroo.
 75. A crocodile.

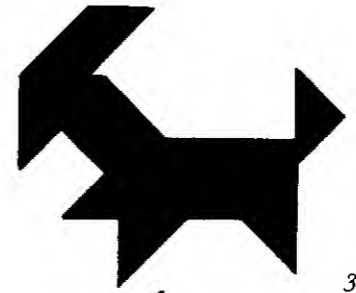
There follow several sea creatures, mainly fish of various kinds, but also

79. A lobster.
 81. A turtle.
 82. A seal.
 83. A shrimp.

THE TANGRAM ZOO



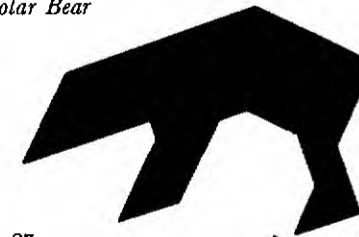
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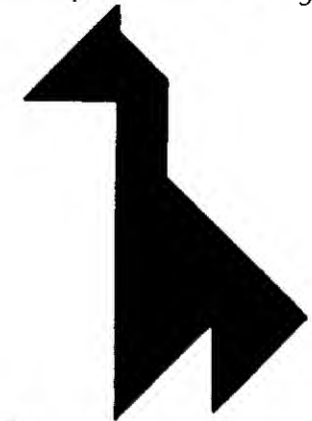
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GOATS

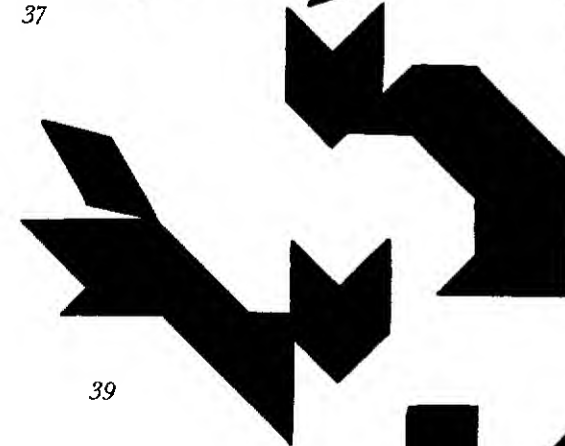
Polar Bear



37

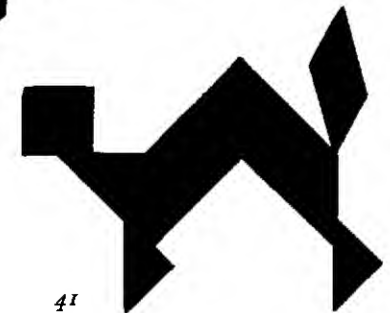


38 Giraffe



39

CATS



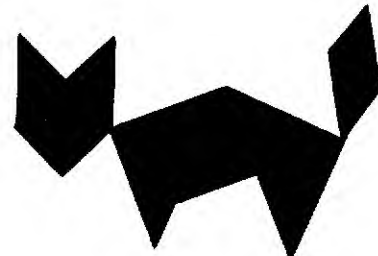
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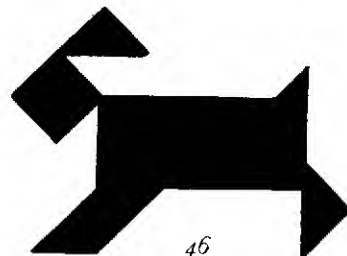
CATS AND DOGS



42



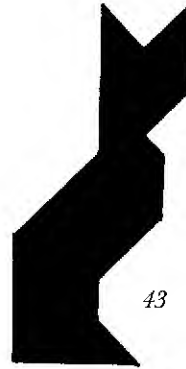
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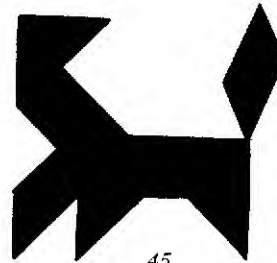
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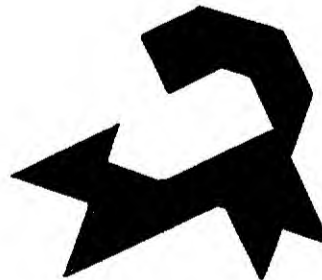
47 Camel



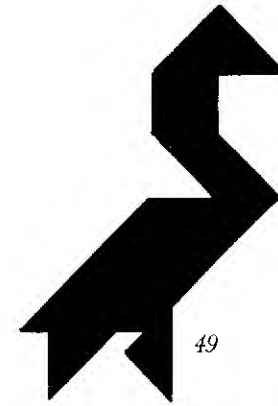
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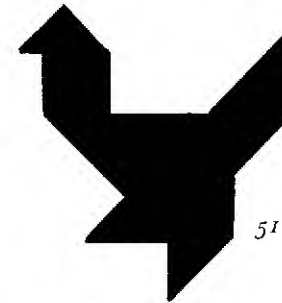
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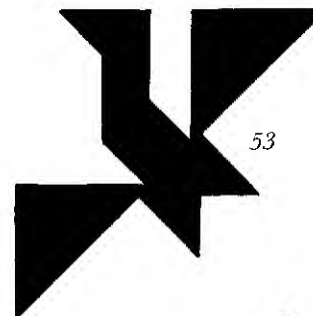
48 Squirrel



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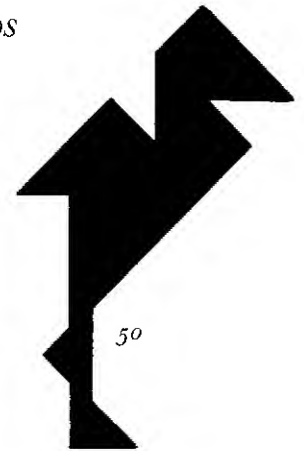


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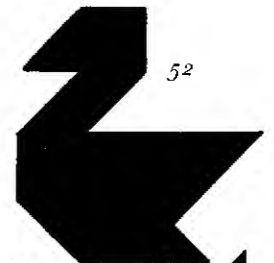


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BIRDS



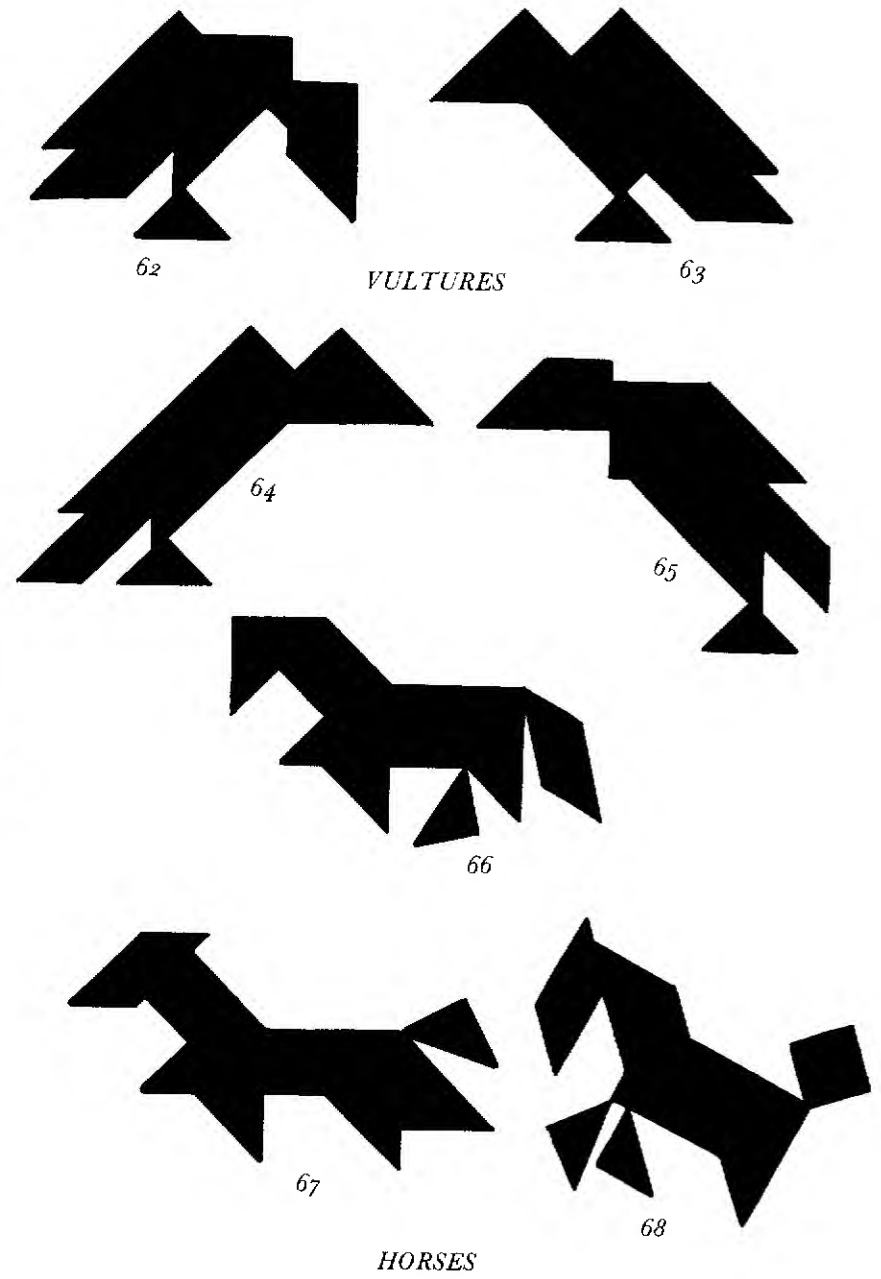
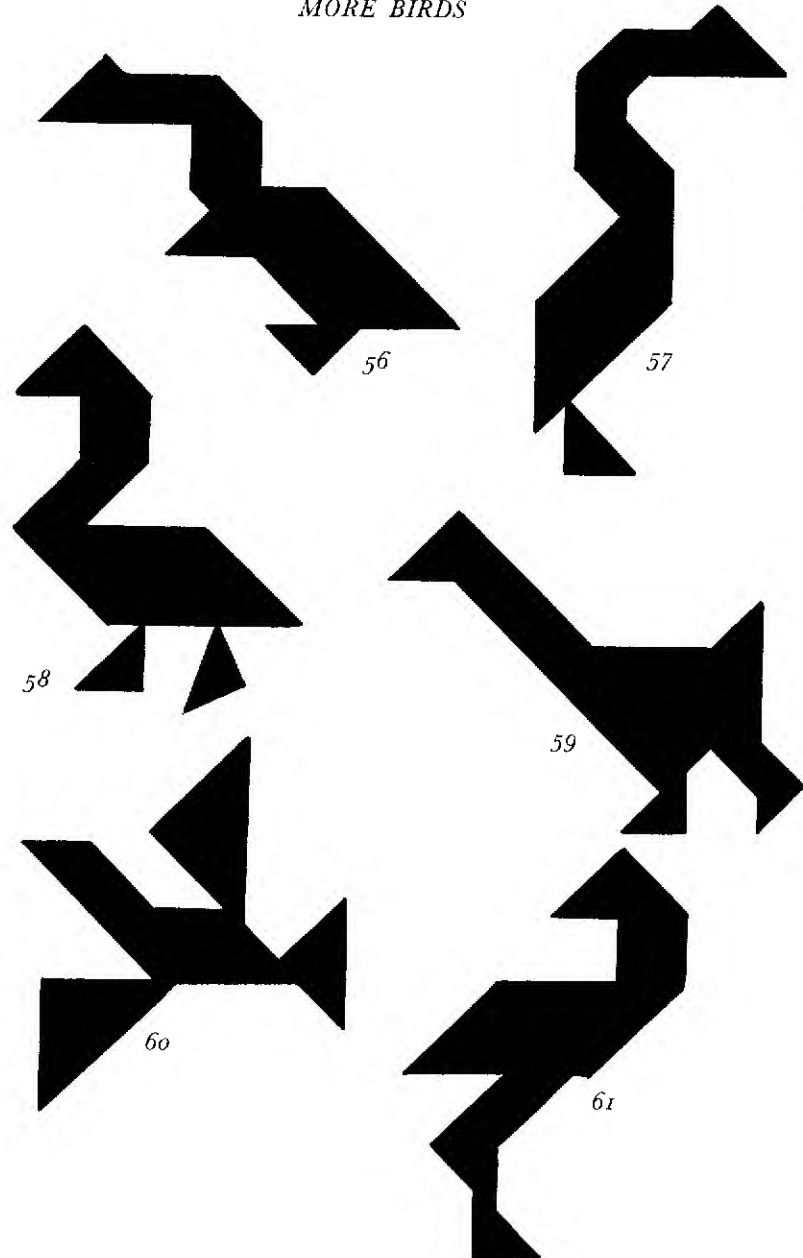
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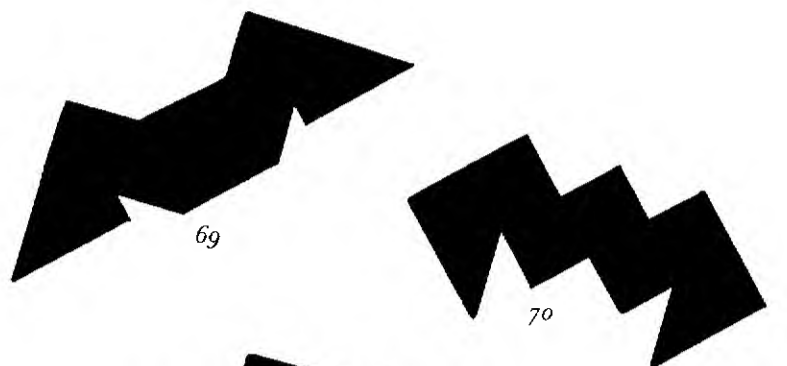


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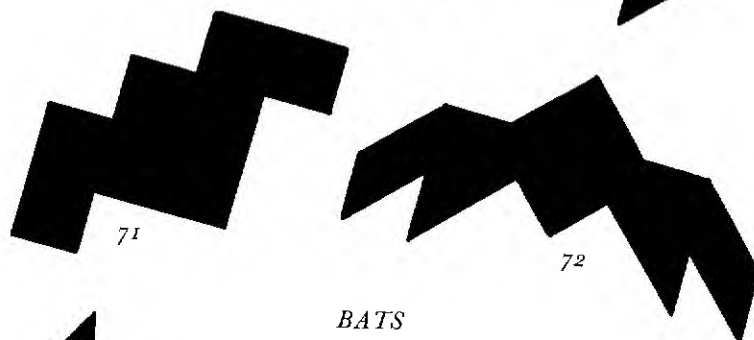
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MORE BIRDS



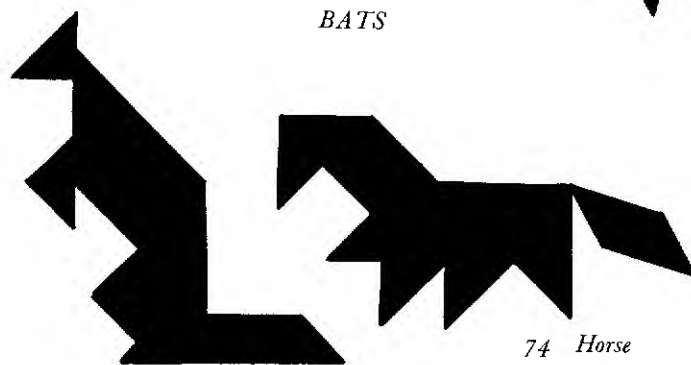
69

70



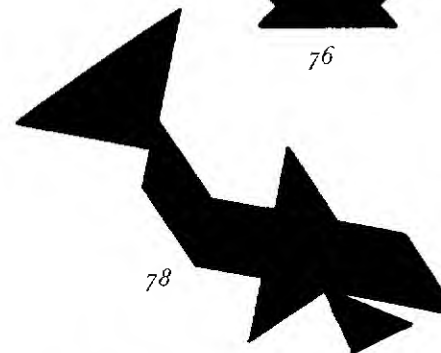
71

72

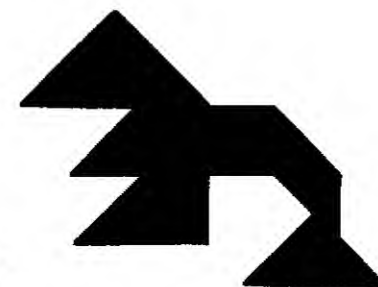
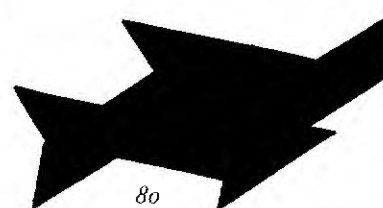
BATS73 *Kangaroo*74 *Horse*75 *Crocodile**SEA CREATURES*

77

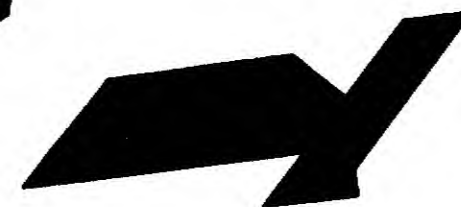
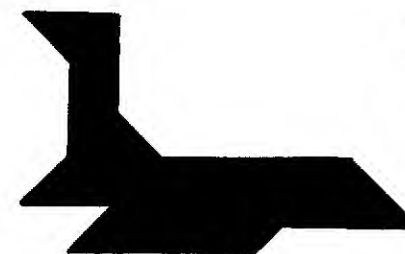
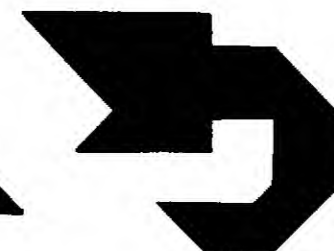
76



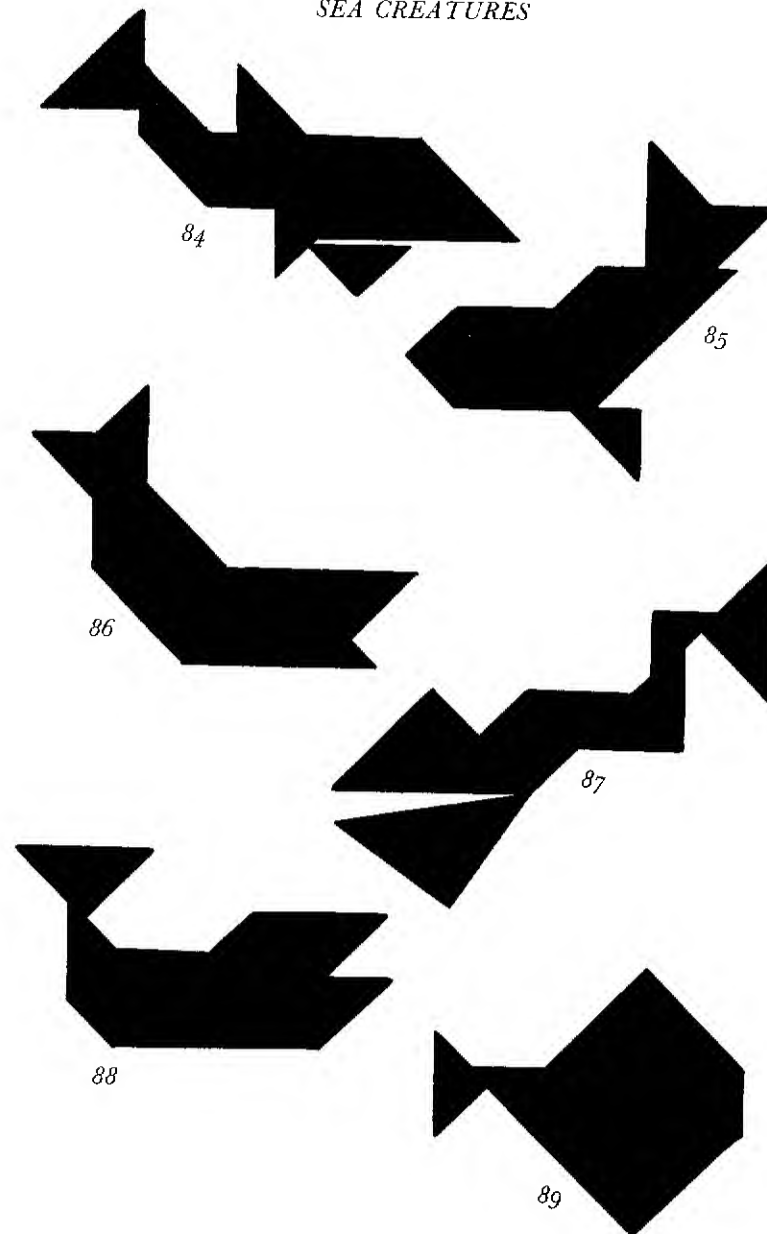
78

79 *Lobster*

80

81 *Turtle*82 *Seal*83 *Shrimp*

SEA CREATURES

3. *Mankind*

WHILE the evolutionist will not deny the propriety of dealing first with animals, we should delay no longer in looking at those tangrams which depict man and his activities. These are among the most interesting of tangrams: for since the human shape is more familiar to us than that of any other object in the universe, we can readily appreciate a tangram which succeeds in portraying the essence of some human form or posture; and just as readily detect the fault in those that do not manage to suggest what they are meant to represent.

Our human parade starts with a shoeshine boy and a customer (Nos. 90 and 91), two men very excited about something, and three acrobats (94, 95, 96), one of whom has just had a fall (or perhaps he is just taking a rest). On the next page are five men who are obviously in a great hurry and a sixth, strolling along with his hands in his pockets, who is clearly in no rush to get anywhere (97 to 102). Then we have four stately medieval ladies (103 to 106) accompanied by a servant (107) and a lady's maid (108).

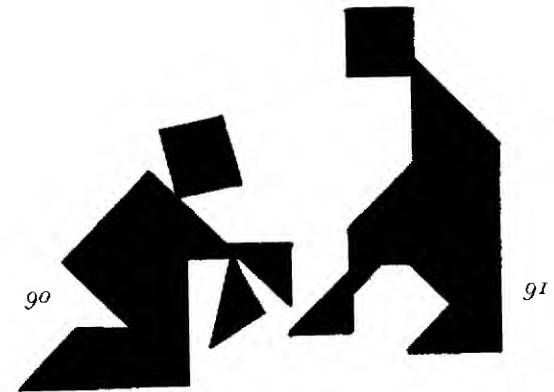
Next we have "Sam Loyd's Portrait Gallery," twenty-six cleverly constructed heads taken from Sam Loyd's book, plus two more given by Dudeney. Sam Loyd gave names to some of these outlines, as follows:

- 120. Old Scotch Piper.
- 121. French Grenadier.
- 122. Colonial General.
- 123. A Turk.
- 124. Aunt Betsy.
- 125. Uncle Rhube.
- 126. Mary Smith.
- 128. John Knox.

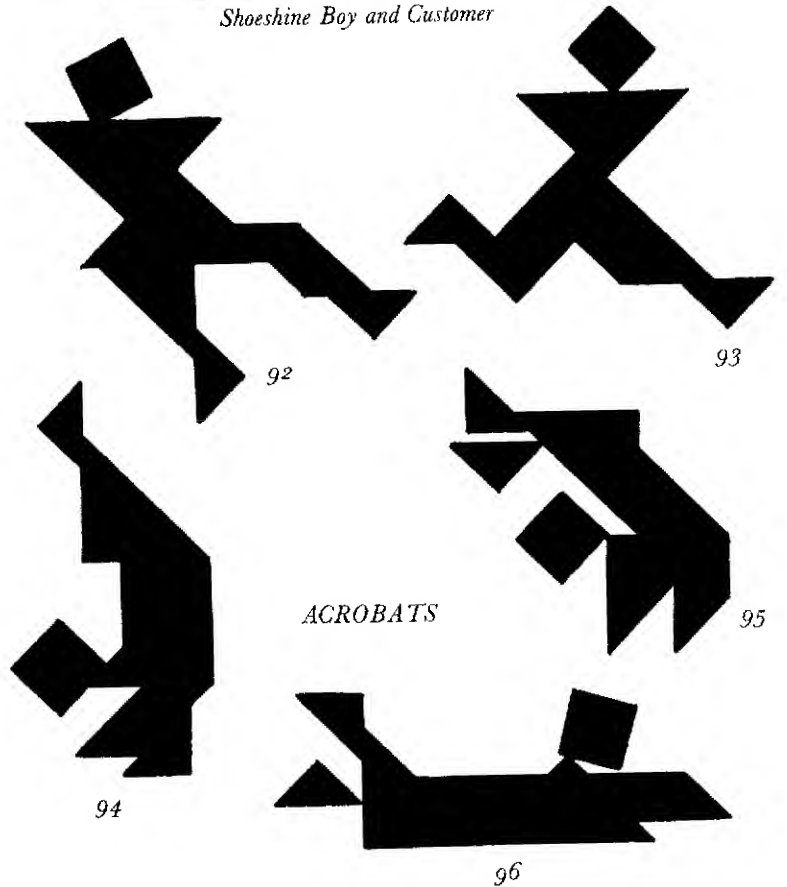
- 129. Tom Sharkey.
- 130. The Professor.
- 131. Buffalo Bill.
- 132. "The Easy Boss."

The Indian chief and his squaw (137 and 138) are also from Sam Loyd's book.

Finally we have four horsemen (139 to 142), a runner (143), and a lady and a gentleman drinking a toast (144 and 145). (Or is he a gentleman? He seems to be seated while the lady is standing—and seated on the floor at that!)



Shoeshine Boy and Customer



ACROBATS

*MEDIEVAL
LADIES*



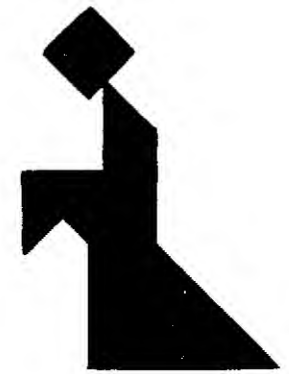
97



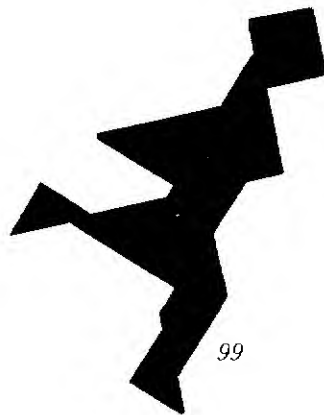
98



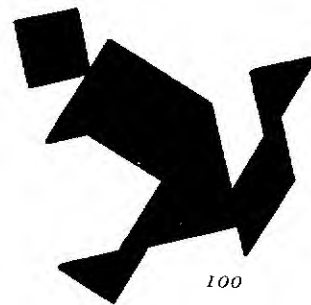
103



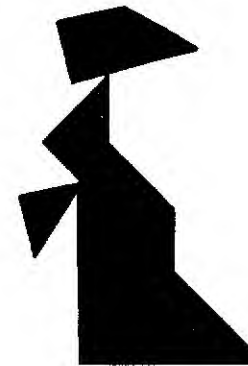
104



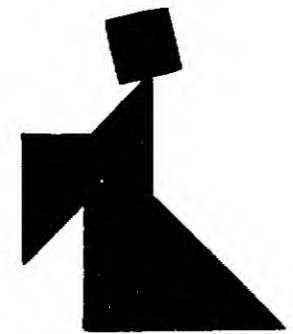
99



100

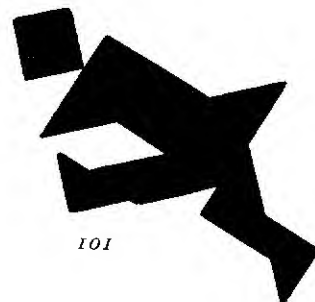


105



106

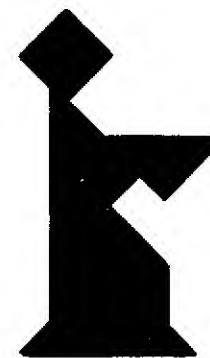
AND SERVANTS



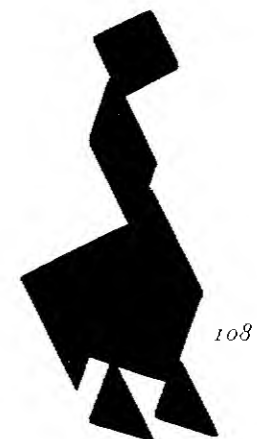
101



102



107



108

SAM LOYD'S PORTRAIT GALLERY



109



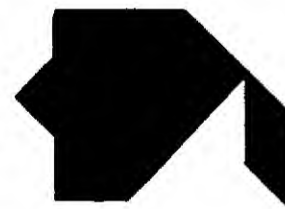
110



111



117



118



119



112



113



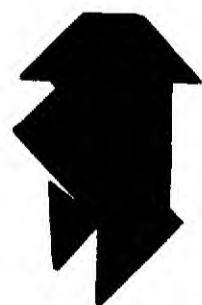
120



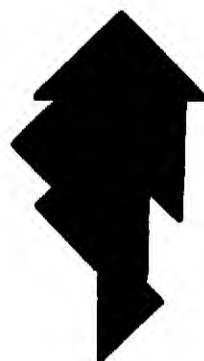
121



122



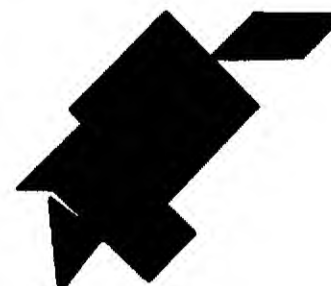
114



115



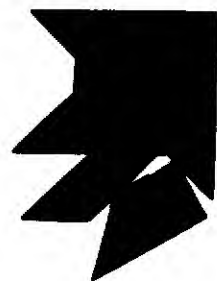
116



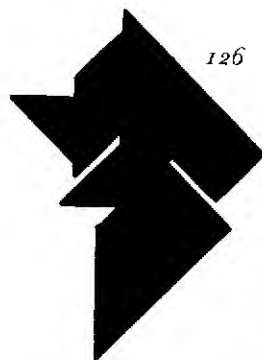
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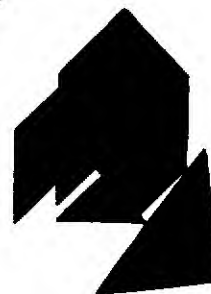
124



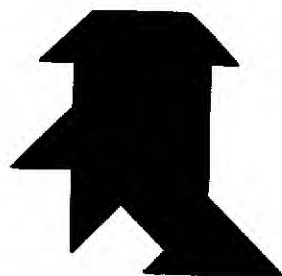
125



126



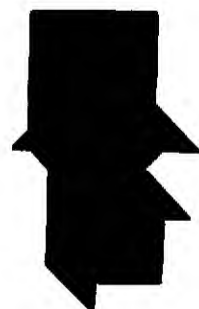
127



128



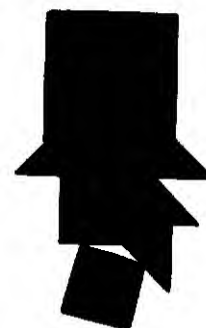
129



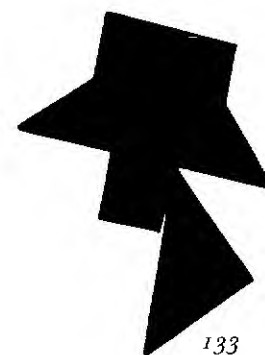
130



131



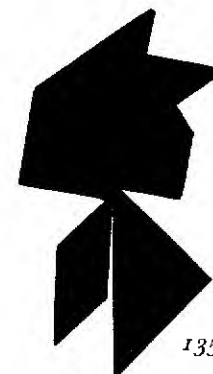
132



133



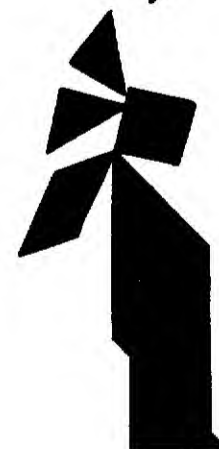
134



135



136

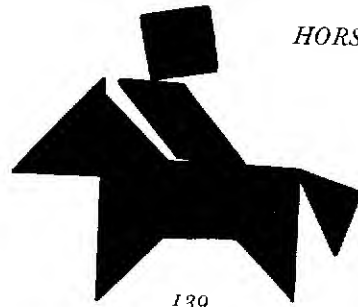


137

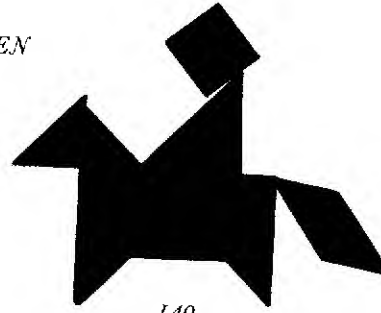


138

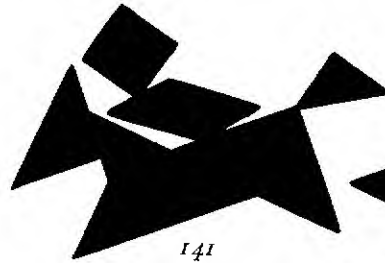
HORSEMEN



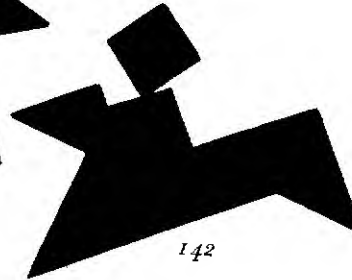
139



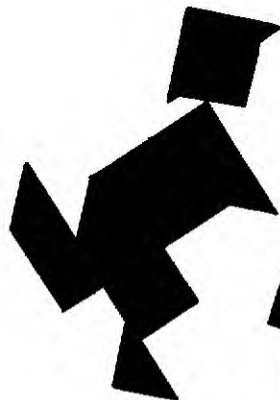
140



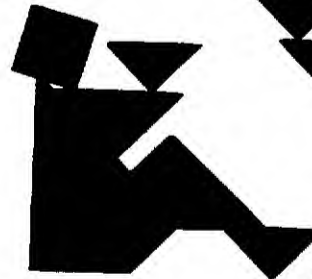
141



142



143



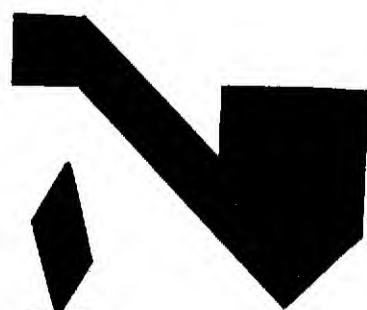
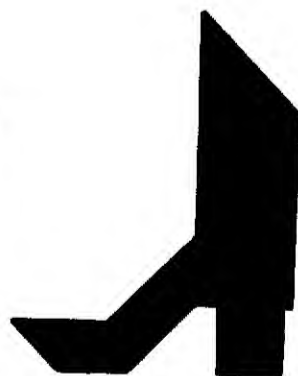
144



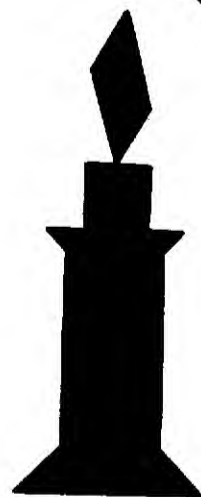
145

4. *Around the House*

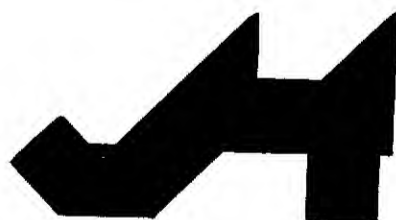
It is only fitting that after looking at man and his activities we should dwell for a few moments on the things that he uses every day. Starting with his pipe (No. 146) we go through a motley collection of shoes, chairs and general odds and ends, ending up with a watering can and a pistol (Nos. 161 and 162). We then have three different kinds of baby carriages—all the latest models and very comfortable, despite the fact that they have square wheels (to which tangram babies have long grown accustomed!). Number 165 is a very superior model which comes complete with a nursemaid (No. 166) to push it. Finally we have Number 167, which is a—now what on earth was that thing meant to be? A saw? A nutmeg-grater? Well, no matter; it is an interesting shape to construct.

146 *Pipe*

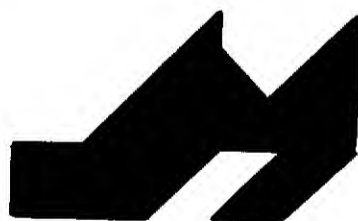
147

148 *Candle*

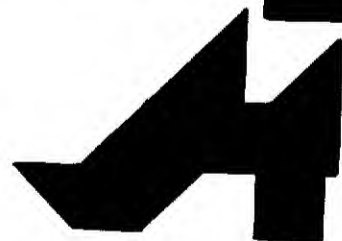
149



150

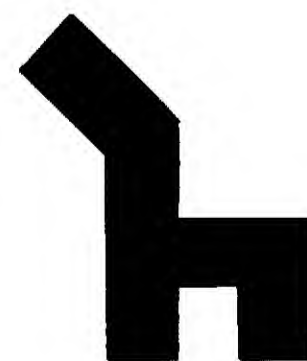


151



152

SHOES

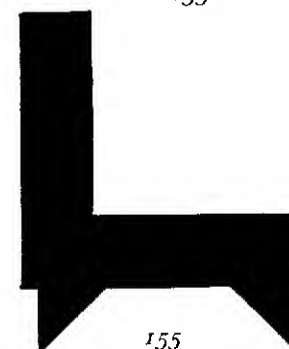


153

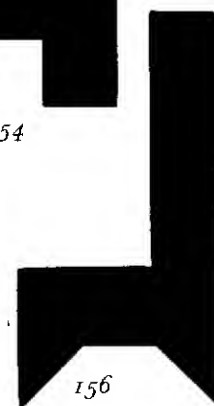


154

CHAIRS



155



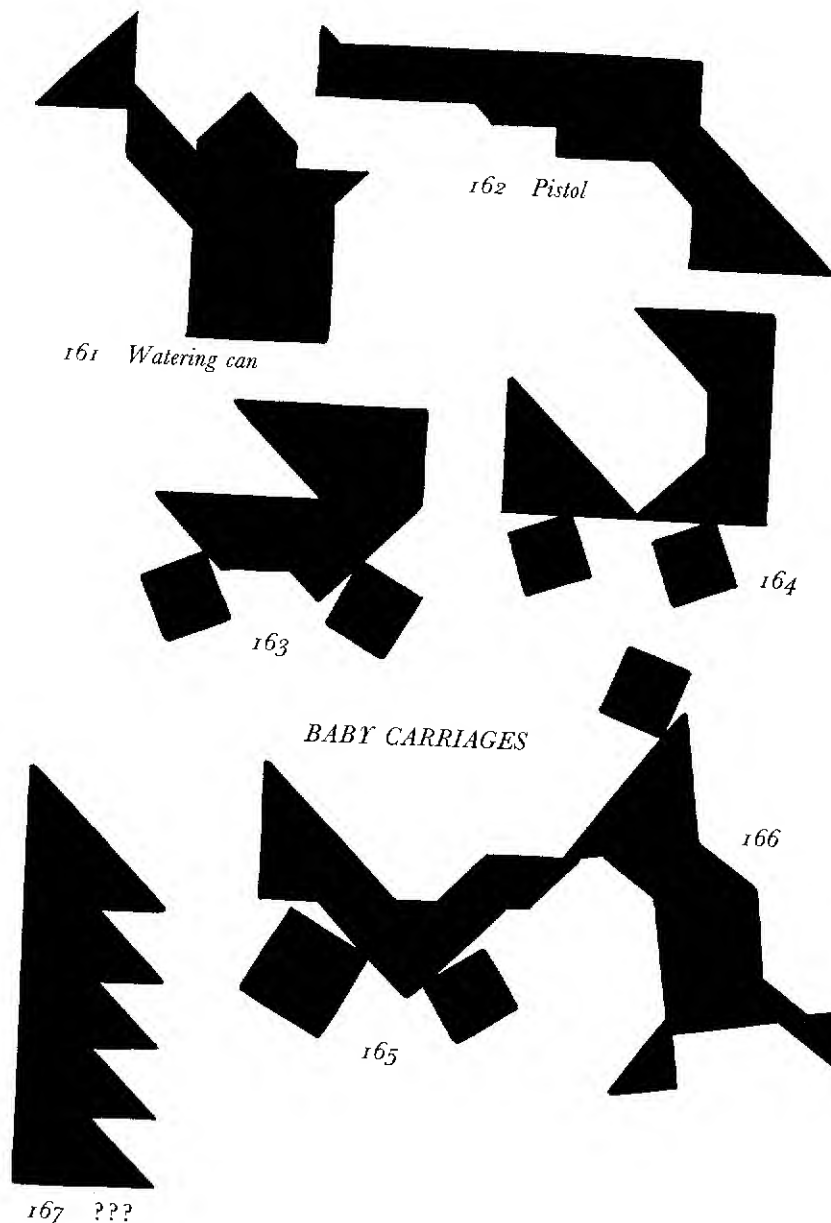
156



157

158 *Shirt*159 *Stocking*

160



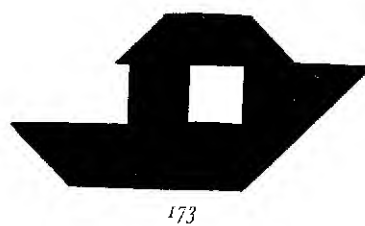
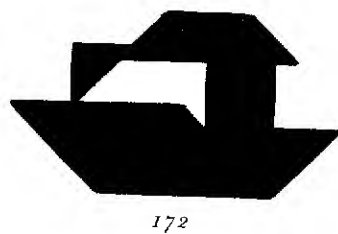
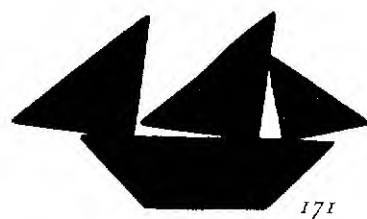
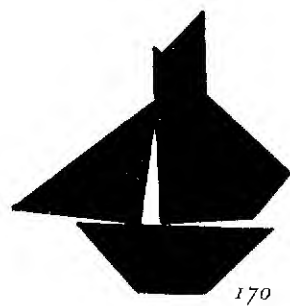
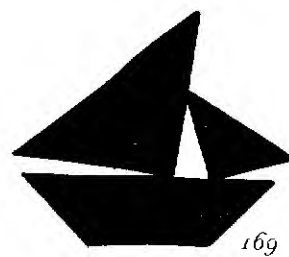
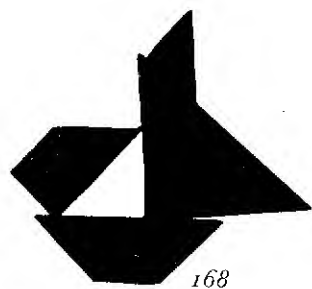
5. Boats and Bridges

THE Chinese books which served to introduce the tangram to the Western world were published in the Treaty Ports of South China, and their pages are full of representations of ships, boats, junks and many other things connected with the sea. It is a curious fact that sailing boats of various kinds seem to be comparatively easy to depict using the seven tangram pieces, notwithstanding the rather complicated silhouettes which they present; whereas steamships, which with their more angular shapes might seem much more suitable for representation by tangrams, are in fact very difficult to concoct. I have been quite unable to produce a convincing tangram of any sort of mechanically propelled craft, with the exception of the launch given in Number 186 (and I won't argue with the reader if he says that this tangram, too, is not very convincing).

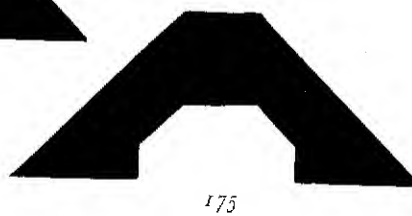
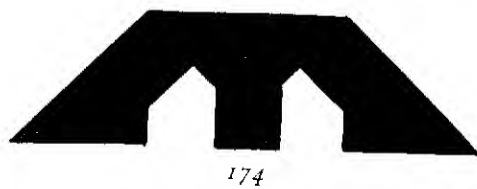
Here, then, is something on which the reader can exercise his creative ability; to produce a reasonably good outline of some kind of steamboat or other modern sea-going craft. It will not be easy!

Along with the ships and boats is given an assortment of bridges, mostly taken from the Chinese books. The last two tangrams of this chapter show a lighthouse and the lighthouse keeper being rowed out to it. (Or maybe it is just a sailor taking his maiden aunt for a row around the bay.)

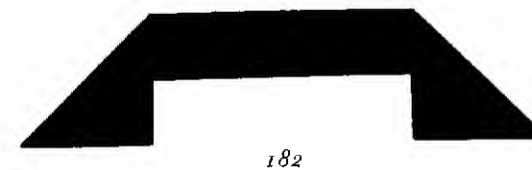
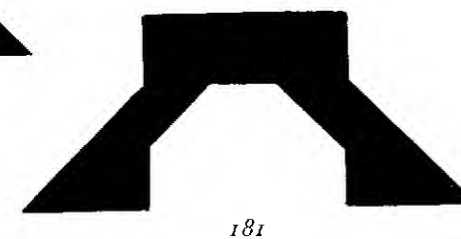
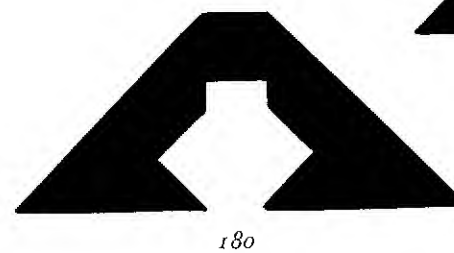
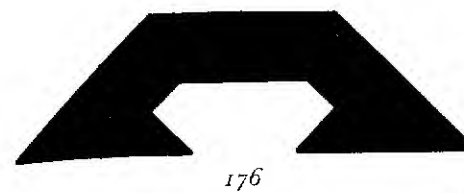
BOATS



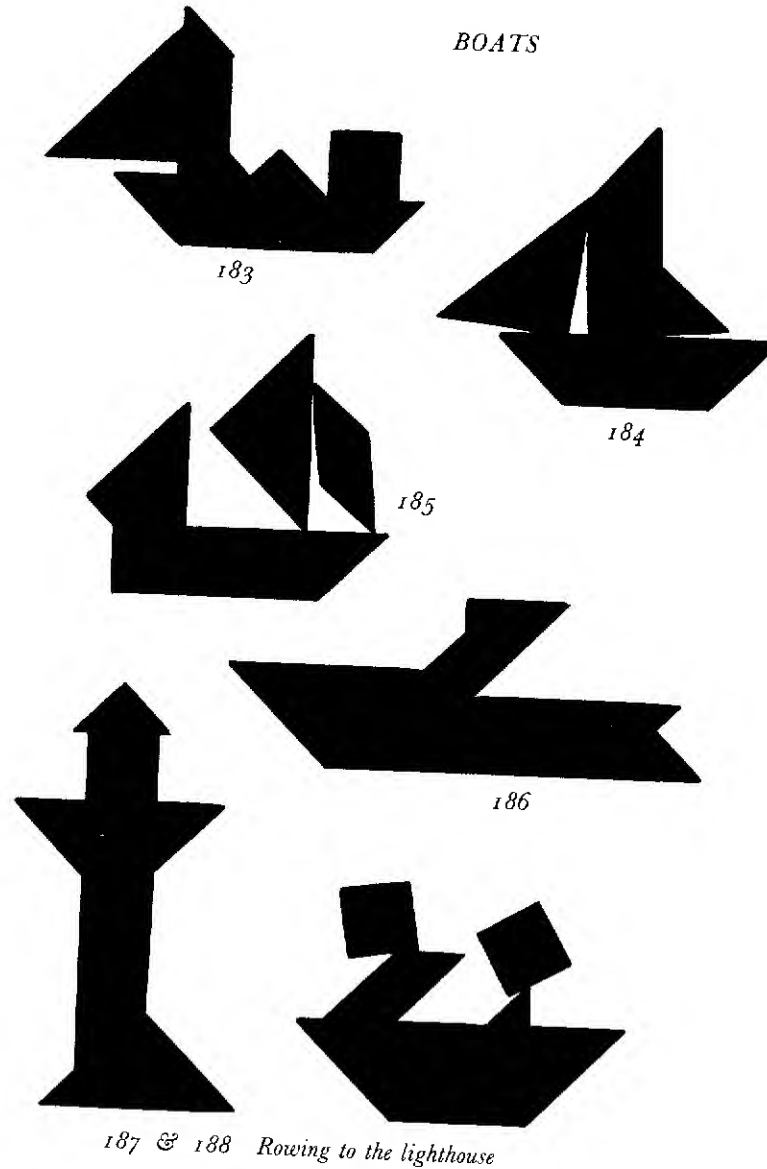
AND BRIDGES



BRIDGES



BOATS



6. Stories and Pictures

Two innovations introduced by Sam Loyd and H. E. Dudeney to extend the scope and interest of the tangram were to construct a series of outlines to illustrate a story, and to bring together several tangram outlines to make a more detailed picture. In this chapter we have two illustrated stories by Sam Loyd, and two pictures by Dudeney.

The first set of tangrams illustrates the well-known nursery rhyme of "The House That Jack Built." In outlines 189 to 199 we see

... the farmer sowing his corn,
 That kept the cock that crowed in the morn,
 That waked the priest all shaven and shorn,
 That married the man all tattered and torn,
 That kissed the maiden all forlorn,
 That milked the cow with the crumpled horn,
 That tossed the dog,
 That worried the cat,
 That killed the rat,
 That ate the malt,
 That lay in the house that Jack built.

To fill out the page, three outlines are given, representing animals that would very likely be found in the vicinity of "The House That Jack Built."

The second story (adapted from Sam Loyd's version) is that of Cinderella. We see Cinderella crying in front of the fireplace (204 and 203); the two ugly sisters (205 and 206), and the fairy godmother with the pumpkin that became a coach and two of the rats that became coachmen (207 to 210). Cinderella dances with the prince (211 and 212), heedless of the clock (213) which is about to strike twelve. The episode of the slipper (214) is too well known to need repetition; everyone

knows the story ends with wedding bells (215 and 216) for Cinderella and her prince.

One outline that clearly ought to have been included in this sequence is Cinderella's coach (when it wasn't being a pumpkin). Sam Loyd does not give one in his book, and I have been unable to devise a satisfactory tangram coach; they all turn out looking like baby carriages! Here again the reader may be more successful. By way of compensation I have included Cinderella's coach among the double tangrams later on in this book.

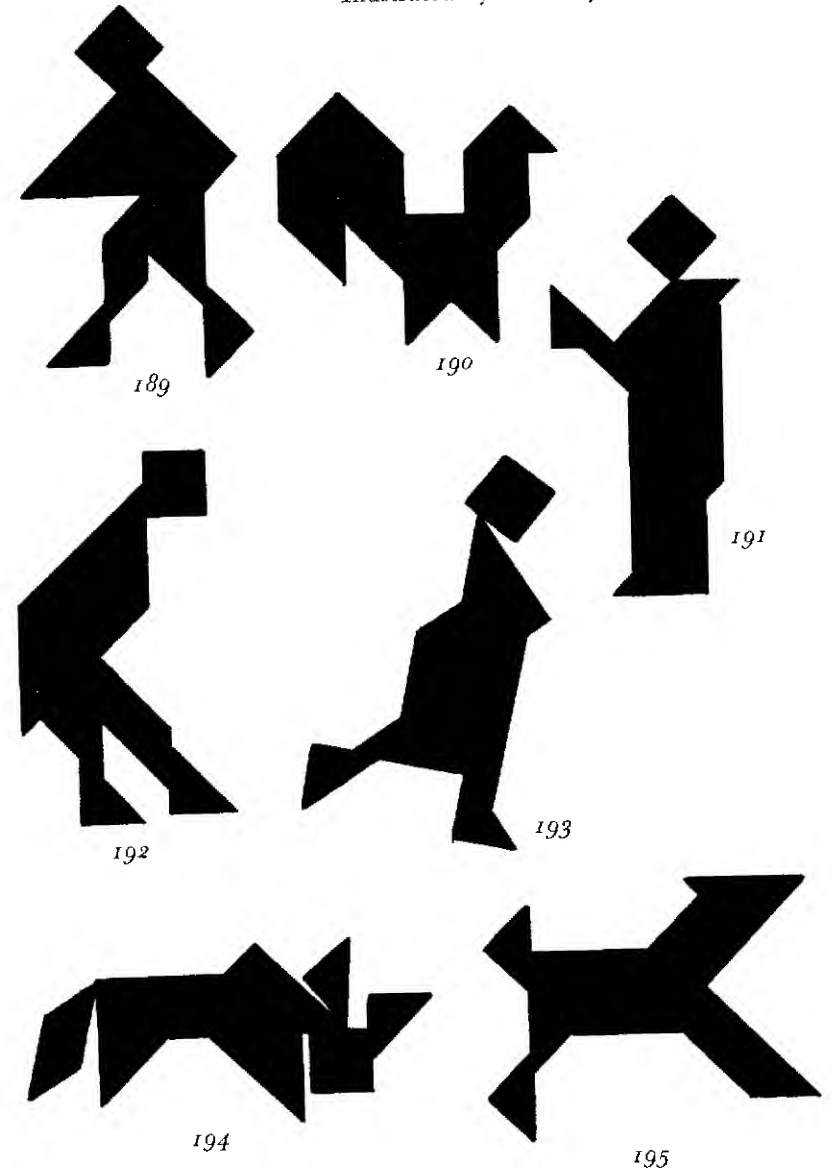
On the next page we have a picture by H. E. Dudeney entitled "A Game of Billiards." Dudeney says, "The players are considering a very delicate stroke at the top of the table."

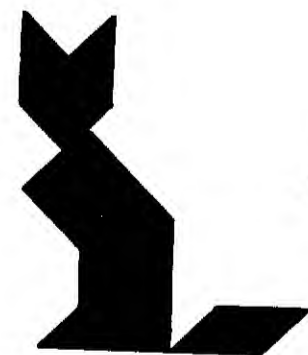
Dudeney's second picture consists of nine outlines (221 to 229) and will not fit on a single page of this book, so has been spread over two. The description of it is best left to Dudeney himself.

My second picture is named "The Orchestra", and was designed for the decoration of a large hall of music. Here we have the conductor, the pianist, the fat little cornet-player, the left-handed player of the double-bass, whose attitude is life-like, though he does stand at an unusual distance from his instrument, and the drummer-boy, with his imposing music-stand. The dog at the back of the pianoforte is not howling; he is an appreciative listener.

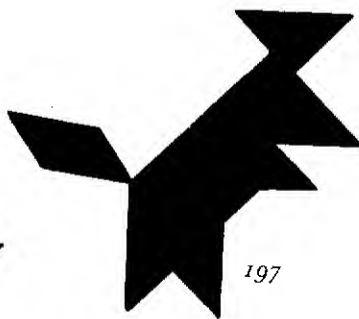
THE HOUSE THAT JACK BUILT

Illustrated by Sam Loyd

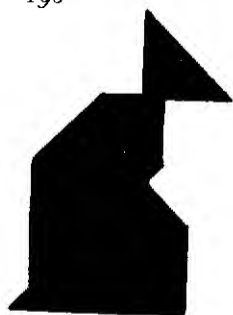




196



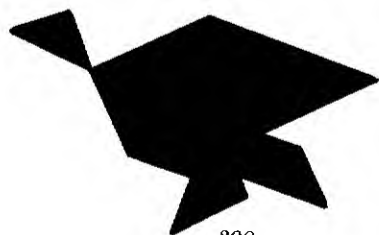
197



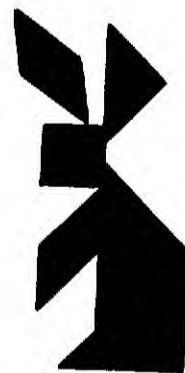
198



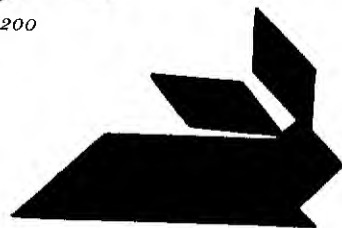
199



200



201



202

THE STORY OF CINDERELLA

Illustrated by Sam Loyd



203



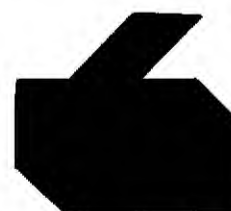
204



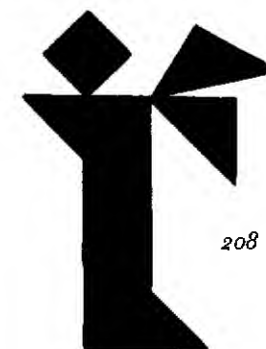
205



206



207



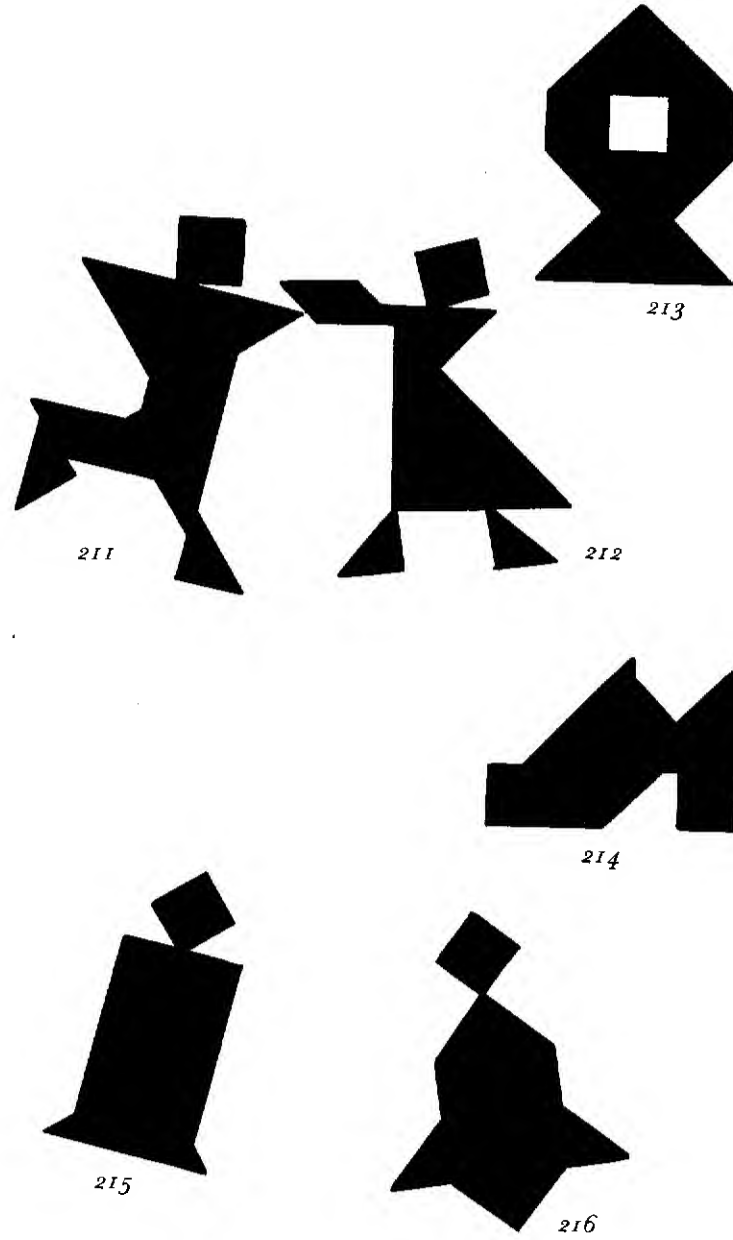
208



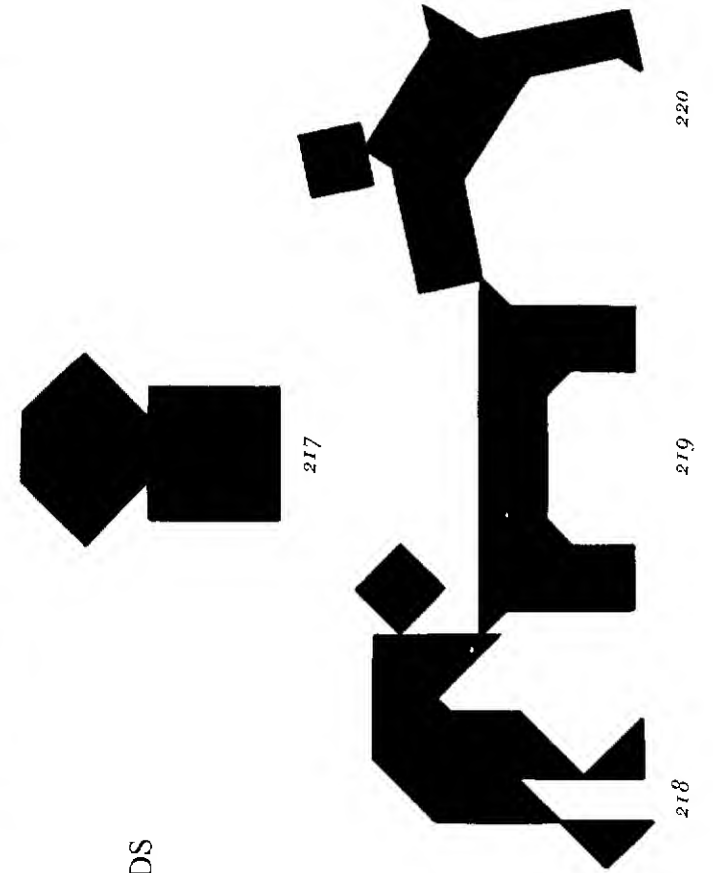
209

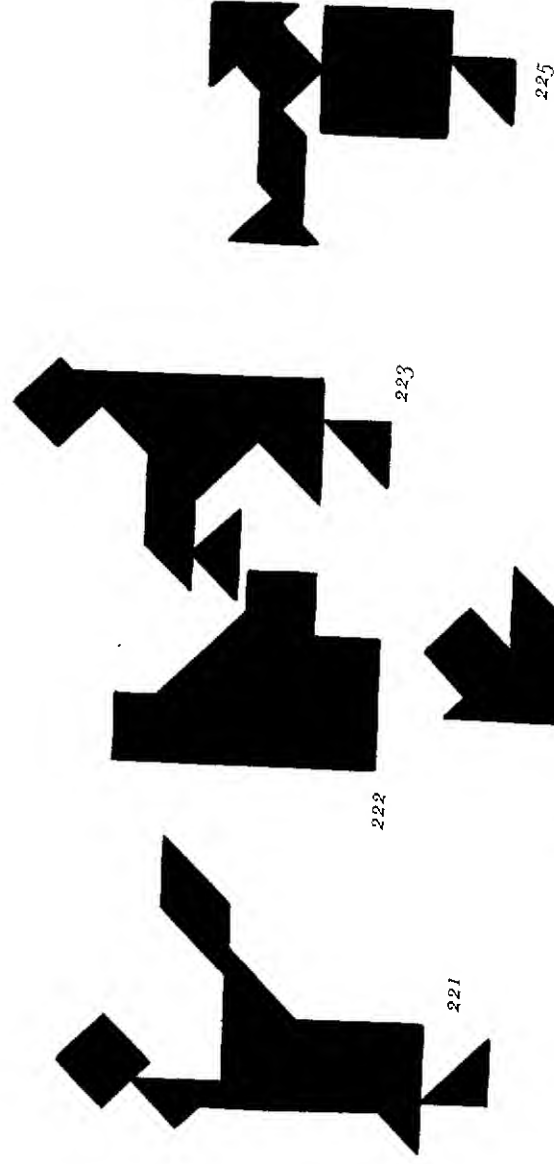


210



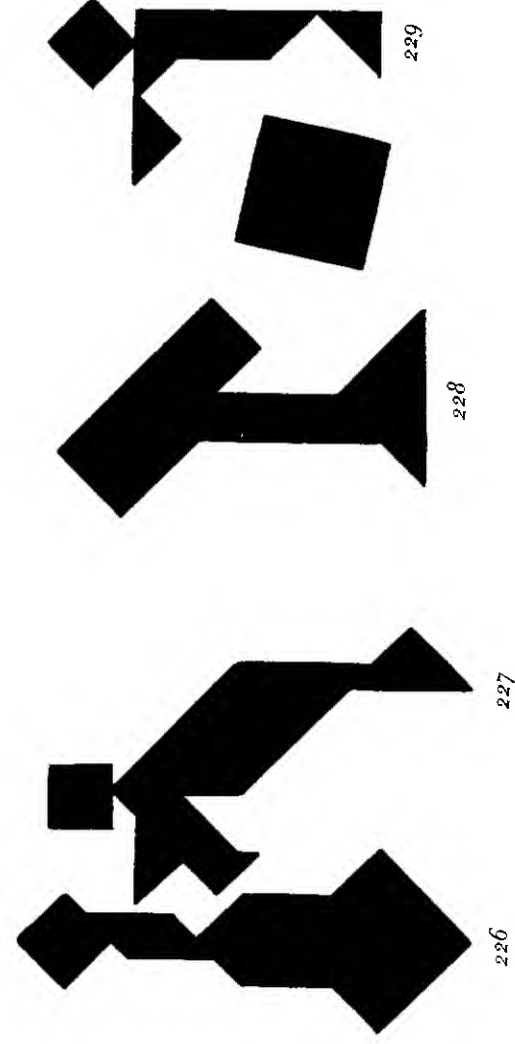
A GAME OF BILLIARDS
By H. E. Dudeney





THE ORCHESTRA

By H. E. Dudeney



7. *A Little Mathematics*

(Don't be alarmed! You will need very little knowledge of mathematics to follow this chapter.)

As we have already remarked more than once, the number of tangrams is very large. This is really an understatement, for the number is infinite! This we can easily see by looking at Number 229, for example. The bottom corner of the square piece that represents the head of the drummer-boy can touch the rest of the tangram at an unlimited number of points along the line that represents the shoulder and the outstretched arm. It is true that the different outlines that one would get by putting this piece in the different positions would not be *very* different each from the other, but in the strictest sense they would have to be counted as distinct.

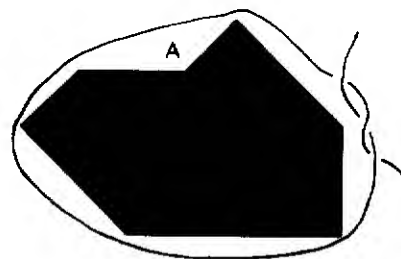
It is a little quixotic, however, to accord the same weight to minute variations in a tangram outline as to the differences between two totally distinct outlines, and one naturally wonders whether, by ignoring trivial variations, one could ask the question, "How many tangrams are there?" with some hope of getting a clear-cut answer. Alternatively one might ask the question, "How many tangrams are there of such-and-such a particular kind."

In 1942, two Chinese mathematicians, Fu Tsiang Wang and Chuan-Chih Hsiung, asked, and answered, the question, "How many convex tangrams are there?" Now, before we go further we must be sure what is meant by "convex" in this connection. Roughly speaking, we can say that a convex figure is one that does not have any recesses in its outline. For angular figures (like tangram outlines) this means that all the angles are less than 180° ; in other words that the corners all stick out instead of in. But the simplest way of seeing the difference between a convex figure and one that is not convex

is to imagine a piece of string or an elastic band pulled tight around the figure, as in Nos. 230 and 231, below. If this causes the string to make contact with the figure all the way round its edge, then the figure is convex; but if there are gaps



230



231

between the string and the edge of the figure, as at A in Number 231, then the figure is not convex, but concave or recessed, where these gaps occur.

Now that we know exactly what “convex” means in this context we can look again at the question asked by the Chinese mathematicians: “How many convex tangrams are there?” One might well imagine that there would be quite a large number of them, but it turns out that there are only thirteen! In the paper in which this result is proved* the thirteen convex tangrams are not drawn, but are merely listed as follows:

Triangles	1
Four-sided figures	6
Five-sided figures	3
Six-sided figures	3

In this chapter we give all thirteen convex tangrams. Some of them are very easy to construct, but a few are rather tricky. The reader may want to try to construct them all without first looking at the outlines given below (Nos. 232 to

* Published in the *American Mathematical Monthly*. Vol. 49 (1942), p. 596.

243). Or he can examine the outlines and then try to construct the tangrams. The total of thirteen is completed with Number 230, but since this tangram was necessarily given in the introduction, it doesn't really count.

AN UNSOLVED PROBLEM

It is clear that convex tangrams are very special indeed; there are so few of them. Can we think of any other special kinds of tangrams that would be more numerous than the convex ones, and yet not infinite in number? As far as I know, no one has ever done so, but I am going to propose a problem of this kind for the benefit of anyone who may feel inclined to tackle it. First we must specify the sort of tangram that we are going to talk about.

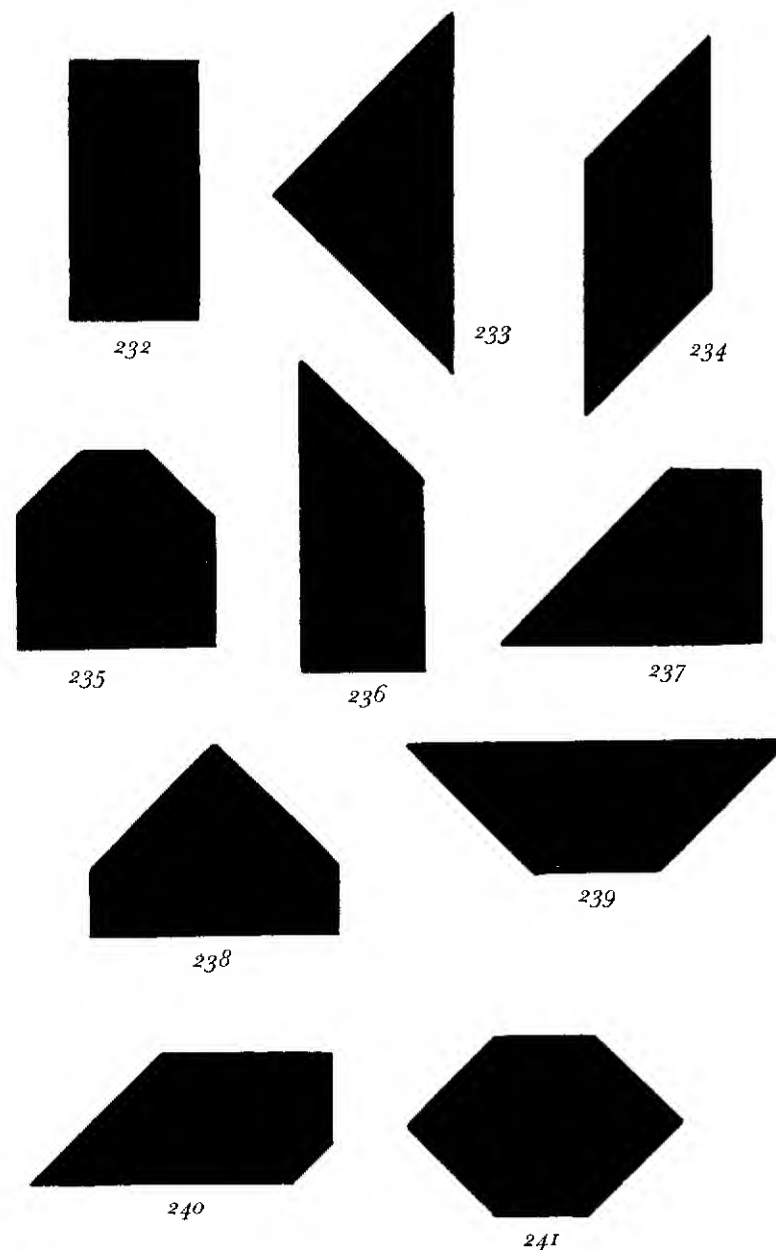
Let us imagine a set of tangram pieces of such a size that the equal sides of the small triangles are 1 inch in length. Then the third side of these triangles will be of approximately 1.414 inches (the square root of 2, to be precise). Now any side of any of the pieces of this set will be one of these lengths, or twice one of these lengths, and we can therefore imagine every side of each of the pieces to be made up of “sections” whose lengths are either 1 inch or 1.414 inches. There will be either one or two sections to each side. In Figure 3 (page 58), which shows the tangram pieces, the ends of the sections are indicated by blobs.

Imagine now a tangram that has been constructed in such a way that wherever two pieces are in contact at all, they are in contact along a whole section of each, so that the ends of these sections coincide. In other words, when two pieces are in contact, the blobs on the two edges will match. This is illustrated by Figure 4. This stipulation on the way in which the pieces are to be placed together considerably restricts the sort of outline that can be produced; on the other hand, large numbers of very interesting tangrams fall into this category. We shall apply one further restriction: namely, that the tangram should be all in one piece. Tangrams which conform to the above restrictions I call “snug” tangrams, because of the close way in

which the pieces fit together. All the convex tangrams are snug; so is Number 231, and so are very many other tangrams in this book. Snug tangrams tend to be rather more difficult to reconstruct from their outlines than tangrams that are not snug, since the close fitting reveals less of the way in which the tangrams have been formed.

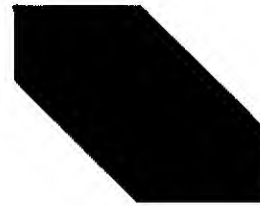
It makes sense to ask the question, "How many snug tangrams are there?" for it can be shown that snug tangrams, unlike tangrams in general, are limited in number—there is only a finite number of them. But how many, exactly? At the moment, nobody knows, and I recommend the problem of calculating this number to anyone who finds it interesting and who has access to a large electronic computer. It is unlikely that the "snug tangram number" will be found without the use of a computer, for it is almost certainly very large, probably well up into the millions, if not considerably larger.

(A preliminary investigation of the problem indicates that it is too complex for the computer to which I have access, but that the problem of programming a computer to find the number of snug tangrams would be a fascinating one—a puzzle that out-tangrams the tangram! Unfortunately, it is a puzzle not available to everyone.)





242



243

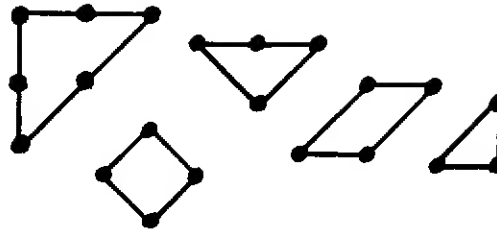


FIGURE 3

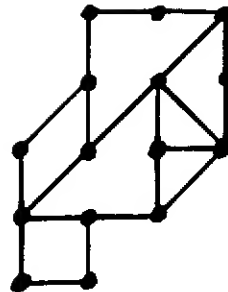


FIGURE 4

8. *Chinoiserie*

As we remarked in the Introduction, the first books of tangrams available in the West were exact reprints of Chinese books on the subject. Some of the objects depicted in these books—ships, birds, and so on—were familiar to Western readers, but the majority of the outlines represented things essentially Chinese, having no Western counterparts. To the editors in England and America these outlines were just so many arbitrary geometrical shapes, and they were treated as such. Yet many of them represent objects of considerable interest, either in themselves or because of their connection with outstanding events in Chinese mythology, literature or history. For this reason we shall look at a few of them in this chapter.

We started this book by forming the letters of the alphabet. The Chinese books did a rather similar thing, but with one important difference. The Chinese schoolboy—pity him!—does not have a paltry twenty-six symbols to learn, as we did, but several thousand characters, many of them very complicated in shape. (To get an idea of their complexity you need only look at our frontispiece, which reproduces the title page of a Chinese tangram book.) Most Chinese characters are far too intricate to be represented in tangram outline, but it is possible to represent some of the simpler ones, and these were not overlooked by the Chinese tangram-puzzlers.

Numbers 244 to 250 give the Chinese characters for “big,” “small,” “six,” “under,” “above,” “of,” and “mountain.” (The characters as usually written are: 大, 小, 六, 下, 上, 之 and 山.) The remaining outlines of this chapter are listed below, with comments where necessary.

251. A goldfish. Goldfish came originally from China and are still extremely popular there, as elsewhere.

252. This is called “riding a crane.” The ancient

Chinese sages, unlike present-day philosophers, had no desire to go traveling about all over the world, and were quite content to stay in one place to meditate. However, when the need did arise to travel from one place to another, they would do so on the backs of cranes. At least, this is what was popularly supposed; presumably, the cranes of those days were stronger birds than they are now!

The tangram represents one of these immortals mounted on his strange steed. The upper parts of the crane and the rider can be seen, but the nether regions are apparently left to the imagination.

253. A bottle gourd.

254. This is a kind of citrus fruit known as "Buddha's hand."

255. A helmet of a kind once worn by Chinese soldiers.

256. This is entitled "Drinking Alone," and alludes to a very well-known poem by the Tang dynasty poet Li Po, "Drinking Alone by Moonlight." The lonely poet invites the moon and his shadow to join him in his solitary tipping.

257. A clock—probably of Western manufacture.

258. A brazier.

259. A dagger.

260. A pinking knife. (Compare the dressmaker's pinking shears.)

261. A teapot—a familiar enough object to us, too.

262. A mulberry hook—presumably for cutting or trimming mulberry bushes and trees. These would be important to the Chinese, since mulberry leaves constitute the staple of the silkworm's diet.

263. This is a raincoat with a hood. As a tangram outline it shows a clever use of empty space.

264. A soup spoon. Any reader who has had a meal at a Chinese restaurant will know the sort of thing that is meant. It is made of china instead of metal, as our spoons are.

265. Another teapot.

266. A Dragon Boat. This refers to a very old Chinese festival, the Dragon Boat Festival, which was held on the fifth

day of the fifth month. Everyone repaired to the river banks, where boats gaily decked out to resemble floating dragons raced up and down the river. The tangram represents one of these boats.

267. The Three-legged Toad. In China, as elsewhere, there are many stories and legends about the moon, and these stories often center around some interpretation of the markings that can be seen on the moon's disk. According to one such legend the markings represent a three-legged toad that lives in the moon. (Rather like our "Man in the Moon.") The tangram is an attempt (not too successful, I fear) to depict this strange animal.

268. A hexagonal pendant. Another tangram that shows good use of empty space.

269. A carp.

270. A flower bowl.

271. A pair of scissors.

272. A saw. (No handles, apparently!)

273. A clothes post. Two of these were used to support a thin bamboo pole on which clothing was hung to dry. This same tangram is elsewhere described as an incense table.

274. A bell.

275. A feather tube. The ceremonial dress of Chinese generals used to include several very long feathers. (These are often very conspicuous in pictures of scenes from Chinese operas, in which generals frequently appear.) When not in use, these feathers were kept in special tubes in order to prevent their being damaged. In much the same way women on both sides of the Atlantic, around the turn of the century, protected the ostrich feathers they wore in their hats.

276. A lucky charm.

277. A look-out tower.

278. A medicine jar.

279. A rack for a hand mirror.

280. A ladle.

281 and 282. Two baskets.

283. This is a stylized arrow with a flag attached, which the Emperor conferred as a symbol of authority.

284. A "foreign candle"; i.e., one imported from the West.

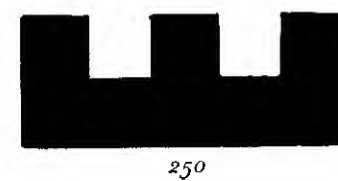
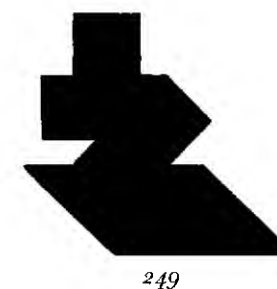
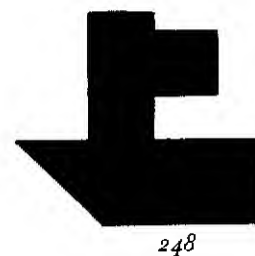
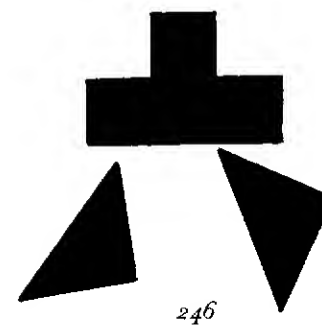
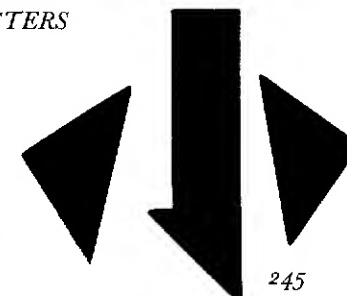
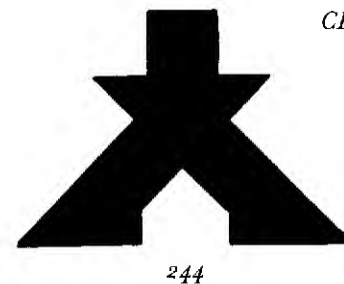
285. A water pot.

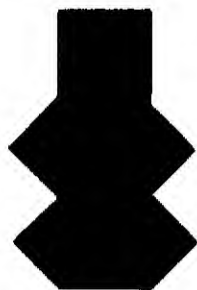
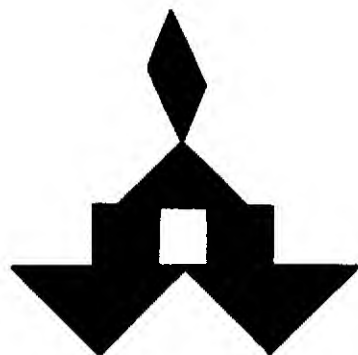
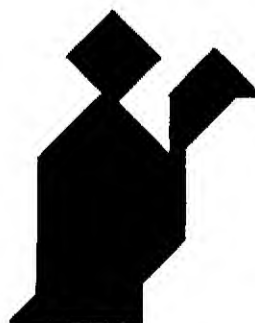
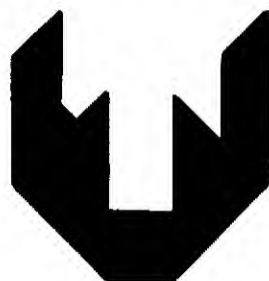
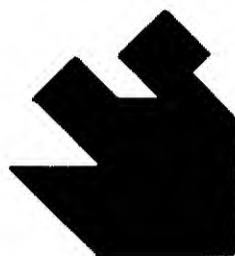
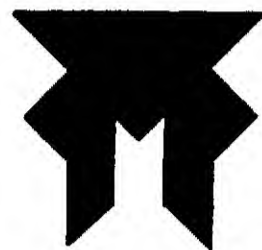
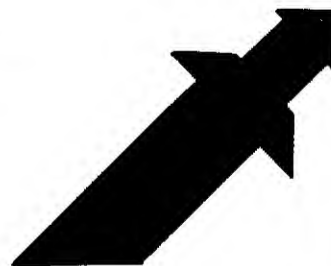
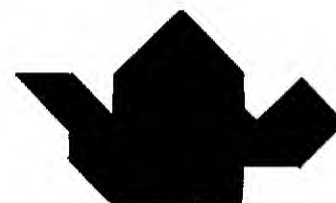
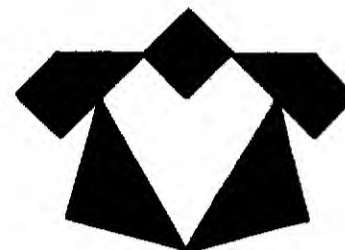
286. A measure.

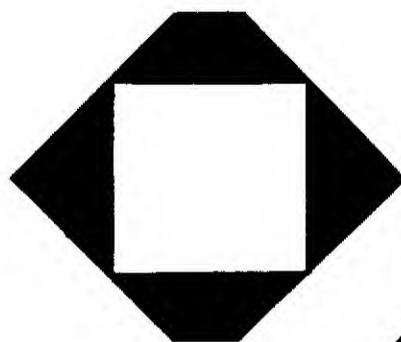
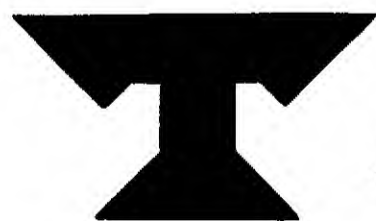
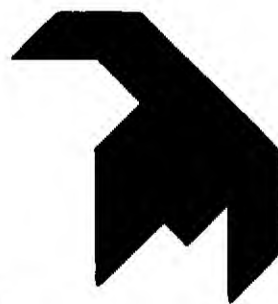
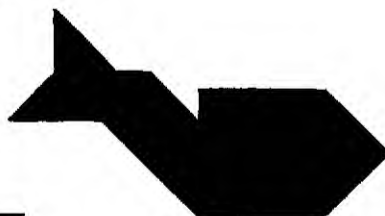
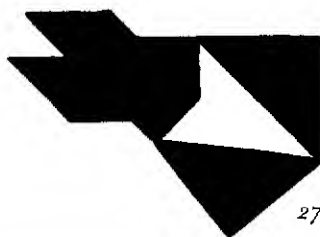
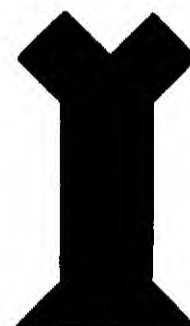
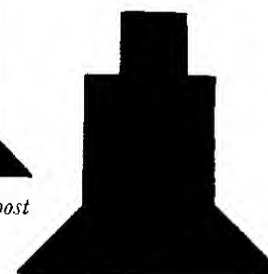
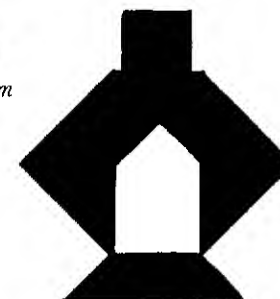
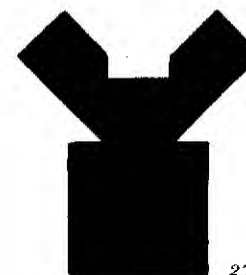
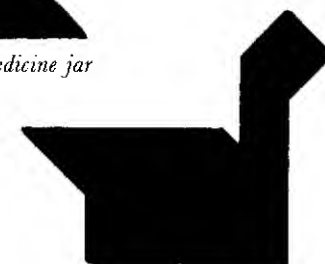
287. A mallet.

288. A Western knife; i.e., a penknife.

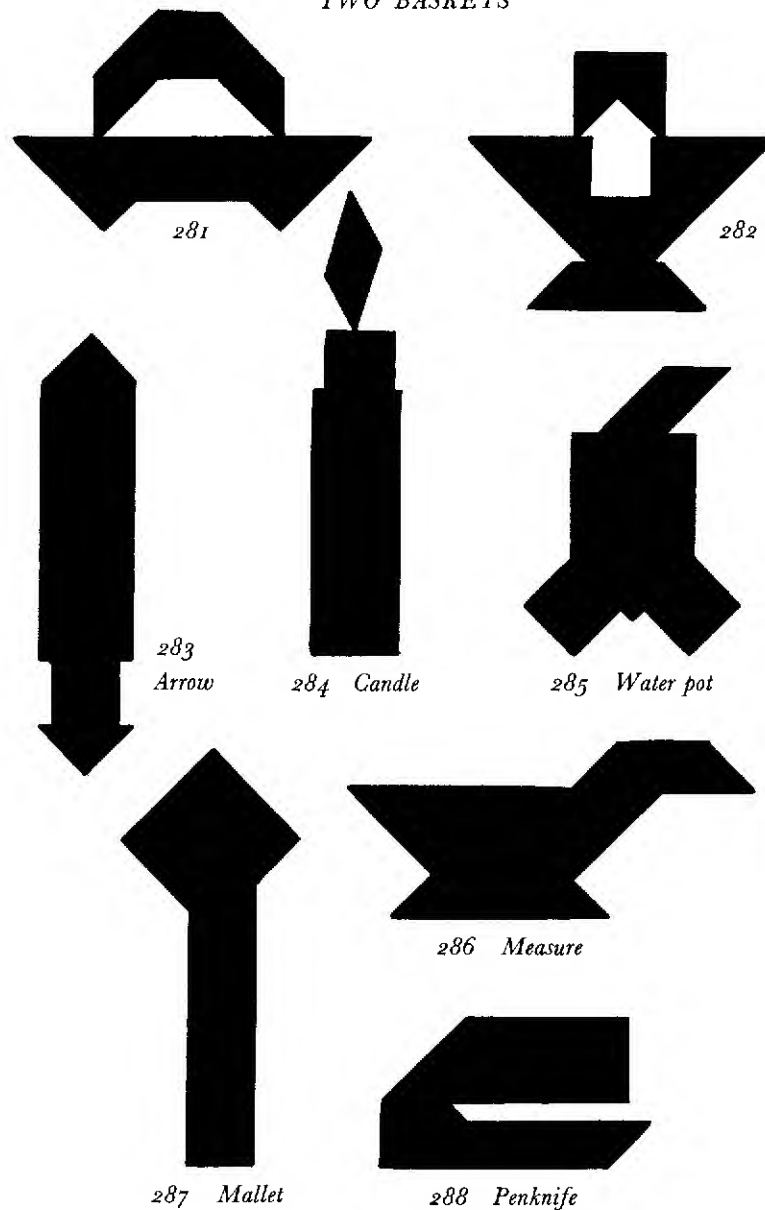
*SOME
CHINESE
CHARACTERS*



251 *Goldfish*253 *Bottle gourd*255 *Helmet*257 *Clock*252 *Riding a crane*254 *"Buddha's hand"*256 *"Drinking alone"*258 *Brazier*259 *Dagger*260 *Pinking knife*261 *Teapot*263 *Raincoat*262 *Mulberry hook*264 *Soup spoon*265 *Teapot*

266 *Dragon boat*268 *Pendant*270 *Flower bowl*272 *Saw*267 *Three-legged toad*269 *Carp*271 *Scissors*273 *Clothes post*274 *Bell*276 *Lucky charm*278 *Medicine jar*275 *Feather tube*277 *Look-out tower*279 *Mirror rack*280 *Ladle*

TWO BASKETS



9. Paradoxes and Illusions

BOTH Sam Loyd and H. E. Dudeney discovered that the tangram could be the source of some pretty paradoxes. The Chinese puzzlers, it would appear, never hit on this particular aspect of the puzzle. Indeed, their diagrams are so carelessly given (many of them being quite wrong!) that it would have been very difficult for the readers of these books to appreciate the subtle differences between tangram outlines on which these paradoxes depend.

Let us start with a paradox by Dudeney. In Numbers 289 and 290 we have two stately gentlemen who appear to be identical in all respects except that one has a foot while the other has not. Yet each of these tangrams uses all seven of the pieces. Whence, then, does the second gentleman get his foot?

This paradox is similar to, and just as effective as, a paradox that has long been known to mathematicians. Take a square whose sides are, say, 8 inches in length, and divide it into four parts as shown in Figure 5, below. These four pieces

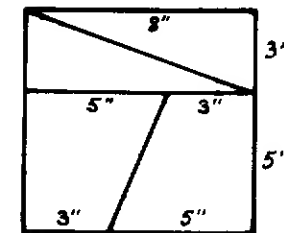


FIGURE 5

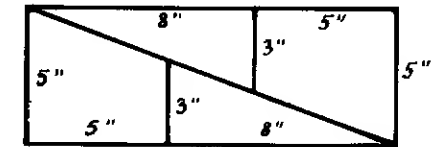


FIGURE 6

can then be rearranged, as in Figure 6, to form a rectangle measuring 5 inches by 13 inches. At once we see the paradox;

for the area of the original square was $8 \times 8 = 64$ square inches, while the area of the rectangle is $5 \times 13 = 65$ square inches. Where has the extra square inch come from?

If the reader does not believe that the four pieces can be fitted together as shown, I suggest he put it to the test by cutting up a square in the manner indicated, and trying the rearrangement himself. An account of this paradox, together with an explanation of it, and many other paradoxes of a similar kind are to be found in *Mathematics, Magic and Mystery* by Martin Gardner.*

There are many other tangram paradoxes of this kind. Numbers 291 and 292 represent two flatirons, each made with all seven tangram pieces. Yet one of them has an extra knob! Again, in Numbers 293 and 294 we have two snuffboxes, and the knob on the second one is twice the size of the knob on the first!

Then we have four vases (295 to 298), all apparently of the same size. Yet only the last is complete. Number 295 has a chip out of the rim, while 296 and 297 each have a hole in the middle, but holes of different sizes! Tangrams 299 and 300 are not very different in outline, but it requires two entirely different arrangements of the pieces to make them.

The reader who has sorted out the difference between Tangrams 289 and 290 (or who has peeped at the Solutions at the end of the book) should have no difficulty in constructing these tangrams, and explaining the paradoxes that they present.

A different sort of paradox, or perhaps "illusion" would be a better word, was invented by Sam Loyd, and is illustrated by the next eight tangrams. They are those used by Loyd in his book. Number 301 is an ordinary wrench; 302 is a rather more elaborate kind of wrench, with a knob to adjust it. The handle is impossibly short, but let's not be too fussy.

Tangram 303 is a Japanese girl in traditional costume, walking gracefully across the floor; 304 is the same Japanese girl executing a polite bow, displaying the bustle which is a characteristic part of the traditional dress. Tangram 305 is yet

* New York: Dover Publications, Inc.

another baby carriage, and 306 a version with large wheels in front.

The reader should try to construct these six tangrams before reading any further.

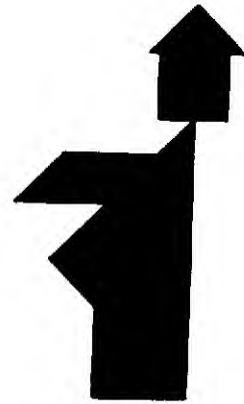
After the reader has constructed these tangrams he should answer (honestly) the following question: Did you notice that Tangrams 302, 304 and 306 are identical, except for the position in which they are drawn?

A similar illusion is presented in Tangrams 307 and 308, showing a man pushing a wheelbarrow. One could very easily overlook the fact that these two tangrams are exactly the same!

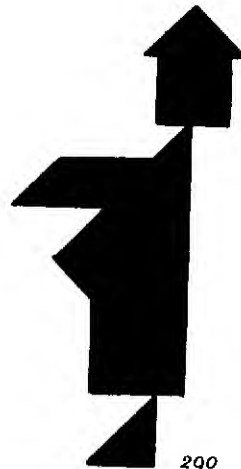
We return to the original type of paradox in Tangrams 309 and 310. One of these pigs has lost his tail. Where did it go?

Finally, from the tangram pieces, which were originally cut from a complete square, we are asked to make a square with a square corner removed from it. The side of the piece removed is one-third the length of the side of the big square. Despite the apparent paradox, this can be done. What is more, the seven tangram pieces can be divided into three groups in such a way that the transition from the complete square to the square-with-a-bite-out-of-one-corner can be made simply by moving these groups *en bloc*. Solve this tangram puzzle and you will have solved the following dissection problem: With two straight cuts, divide a square into three parts which can be rearranged to form a square with a square portion (of dimensions given above) missing from one corner. These two cuts are indicated by extra heavy lines in the Solutions section at the end of this book.

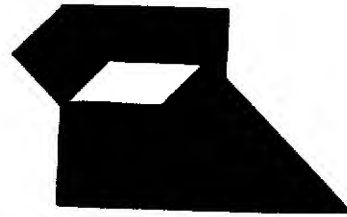
TANGRAM PARADOXES



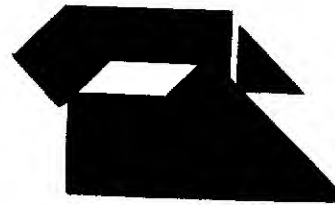
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290



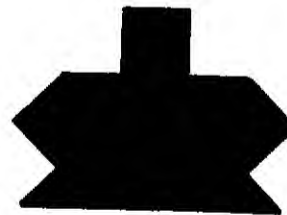
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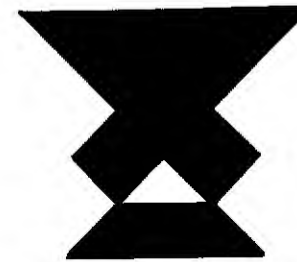
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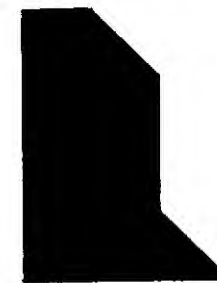
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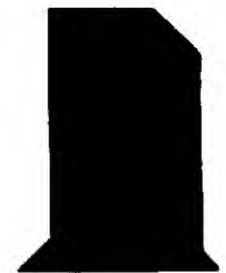
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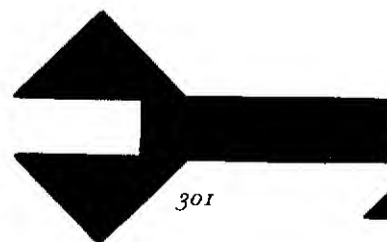


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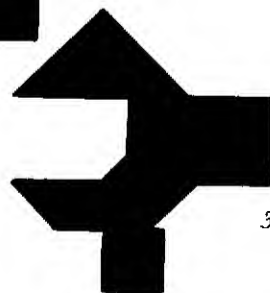


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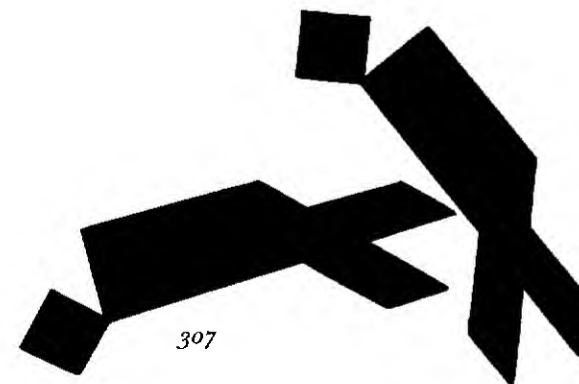
ILLUSIONS



301

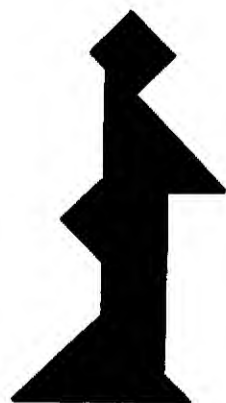


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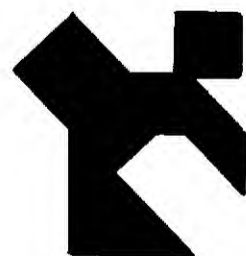


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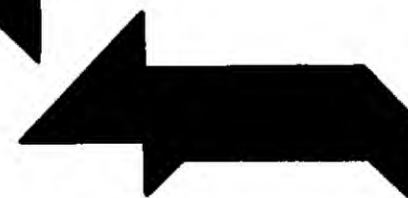
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304



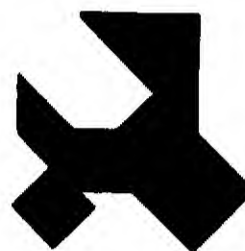
309



310



305



306



311

10. *Double Tangrams*

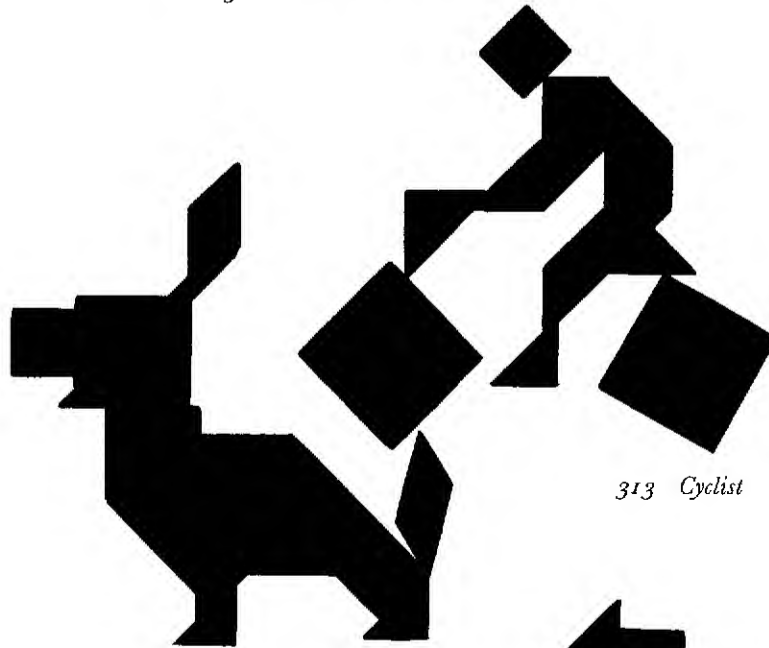
TANGRAMS are amazingly varied and numerous, but there are, of course, limits to what can be done with only seven pieces. It is natural, therefore, to try the effect of using two or more sets of tangrams in an endeavor to achieve more intricate results. Dudeney's pictures (Chapter 6), even though they use several tangram sets, are nevertheless in the true tangram tradition, since each element in the picture is a single complete tangram. In this chapter we shall look at some "double tangrams," made from two sets of tangram pieces, in which the fourteen pieces are used all together, with no separation between the pieces of one set and those of the other.

As the reader will see from the outlines given in this chapter, much more intricate shapes are possible with a double tangram set; yet, on the whole, these double tangrams are less interesting than their single brethren. With fourteen pieces to play around with, one cannot help but feel that it should be possible to arrive at a reasonable likeness to just about anything. Consequently, the sense of achievement that one gets on producing a recognizable cow, sailing boat, human figure, or what have you, from a mere seven pieces, is quite lacking. For all that, the double tangrams are not altogether without interest, and certainly deserve mention. In the following pages we give a selection of double tangram outlines.

Numbers 312 to 317 are self-explanatory (I hope). Number 318 is Cinderella's coach, which we promised in Chapter 6. (If you think it still looks like a baby carriage, I can only say, "Go and make a better one, then!") Then we have a space rocket and a television set (We're really up to date now!), followed by a locomotive. Finally, a microscope (322), a telescope (323), and that most fiendish of all inventions of the devil, an alarm clock!

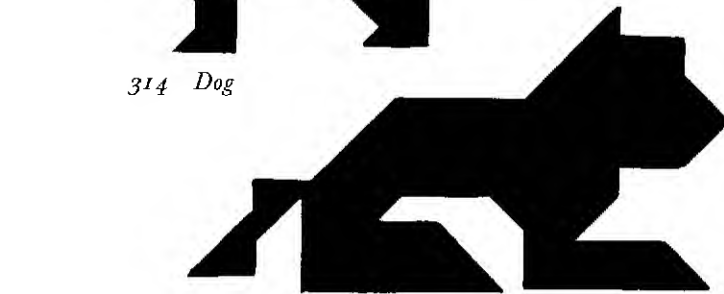


312 House

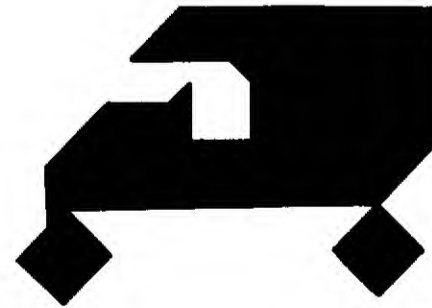
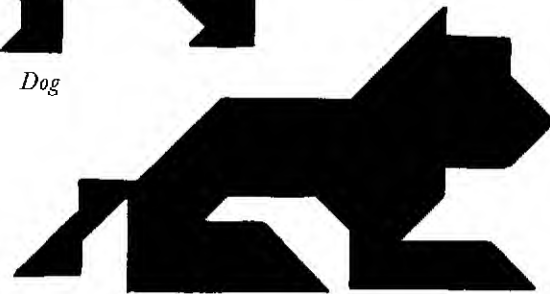


313 Cyclist

314 Dog



315 Lion

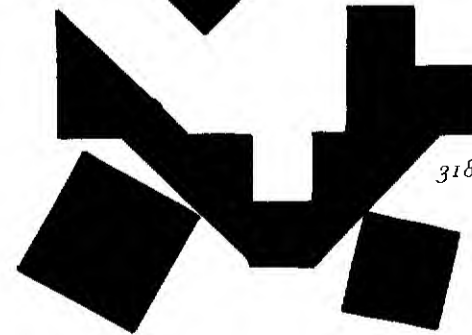


316 Car (ancient vintage)

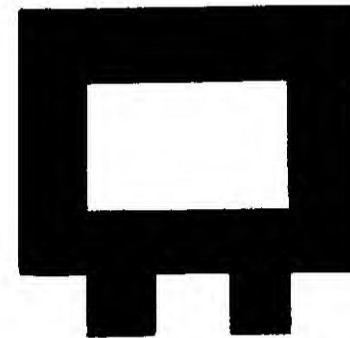
317 Sports car



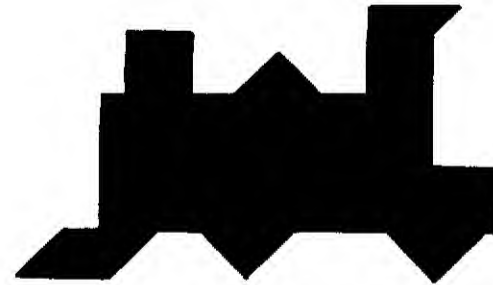
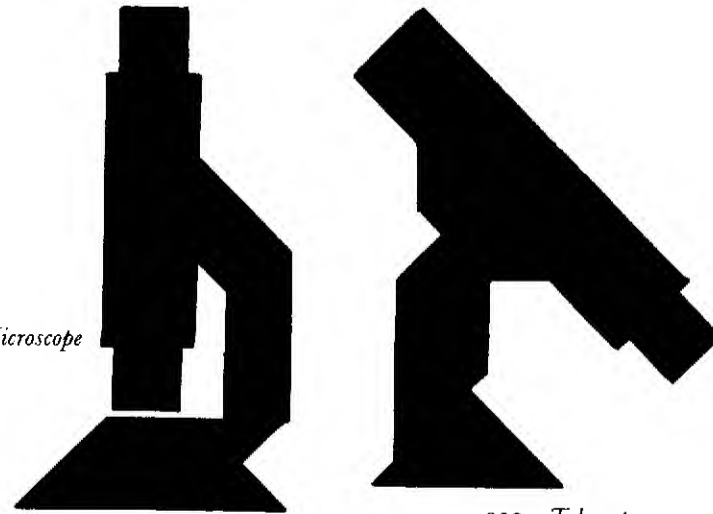
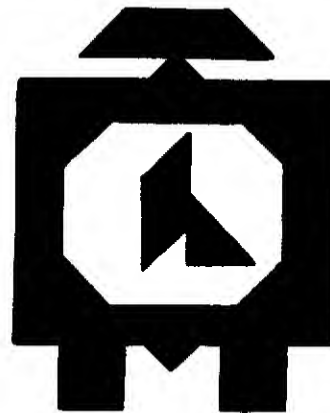
318 Coach



319 Rocket



320 Television set

321 *Locomotive*322 *Microscope*323 *Telescope*324 *Alarm clock*

11. The Fifteen-Piece Puzzle

IN 1962, while searching in the British Museum for further information on tangrams I came across two volumes printed in China in the latter half of the nineteenth century dealing with a puzzle which will be of interest to those who find the tangram interesting. These books were written by one T'ung Hsieh-keng, and the Chinese name for the puzzle is literally "Increase-knowledge puzzle"; we shall call it the fifteen-piece puzzle. As this name suggests, fifteen pieces are used, and they are cut from a square in accordance with the following figure:

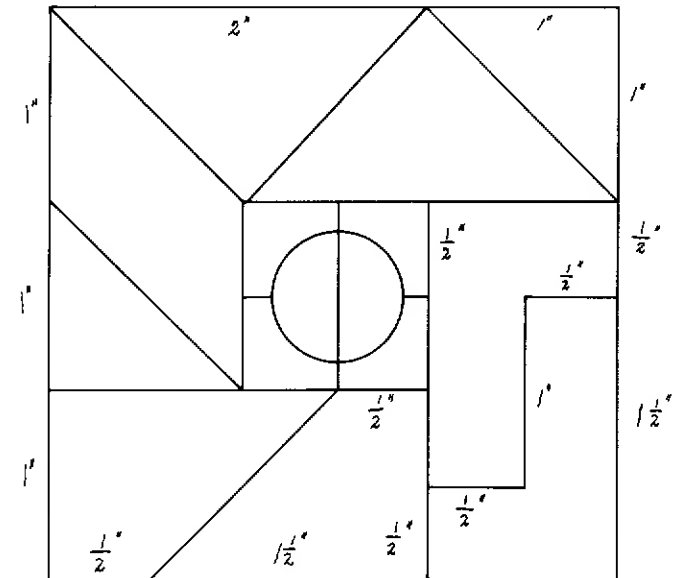


FIGURE 7

Suggested measurements are given, though the square can be of any size. With these measurements the diameter of the

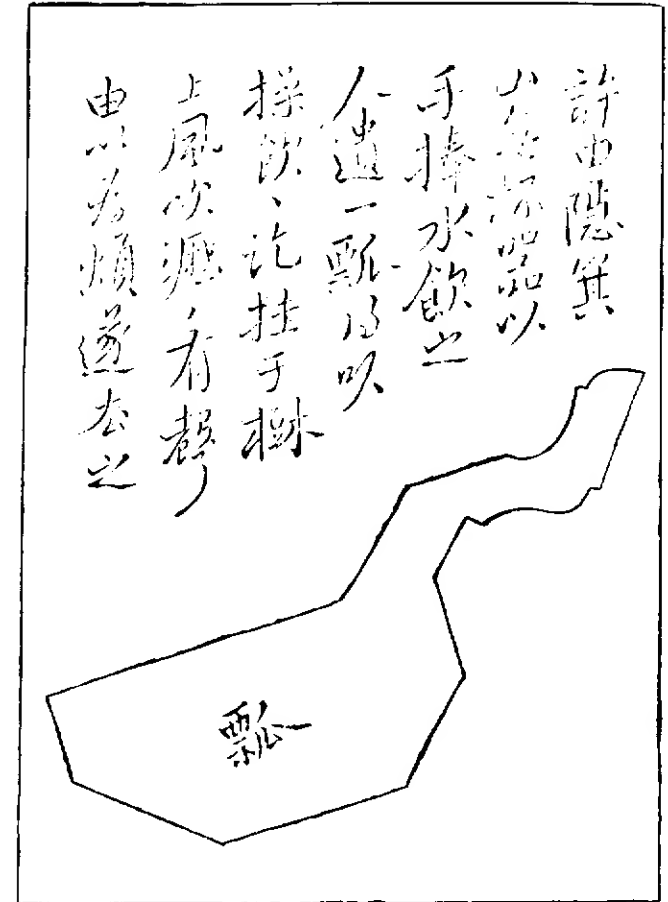
central circle is $\frac{3}{4}$ inch. As with the tangram pieces, all the angles are either 45° or 90° or 135° .

What was said in the last chapter about the lesser interest of the double tangrams, compared with single tangrams, will apply equally well here; with fifteen pieces one ought to be able to make a reasonable outline of almost anything. The fifteen-piece puzzle has some added points of interest, however, in that the pieces are more varied in shape; there are even some curved edges.

The two Chinese volumes contain a large number of pictures made up from the fifteen pieces. Many of them are straightforward outlines of single objects, not so very different from the tangrams or double tangrams given in this book; but many of them are complete pictures, very much in the traditional style of Chinese painting, and with a typically Chinese choice of subject matter—a pavilion in the moonlight, a man asleep in a drifting boat, and so on. As is typical in Chinese paintings, an appropriate poetic quotation is written in one corner of the picture.

Unlike the diagrams in the Chinese tangram books of the early nineteenth century, the illustrations in these two volumes are carefully and pleasingly drawn. For this reason the outlines appearing in the next few pages are given exactly as in the original. They are representative of the many delightful constructions that these two books contain.

The reader who has derived pleasure from trying to make the tangrams in the first ten chapters will surely want to cut himself a set of fifteen pieces and endeavor to reproduce these pictures. In some of the solutions of these puzzles given at the back of the book the relative positions of the various parts of the picture have been altered for the sake of convenience; the construction of the individual parts, however, is correctly given.



瓢區其腹
 錢大其口
 斟之酌之
 呼者陶友



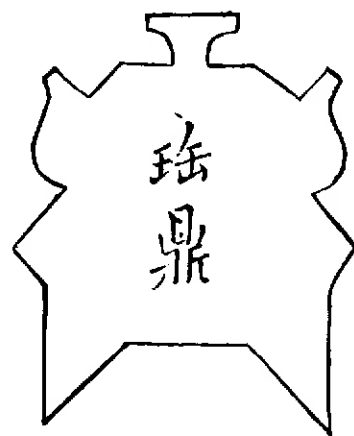
326

不炊而沸
 不汲自盈
 網緼之氣
 自然所生



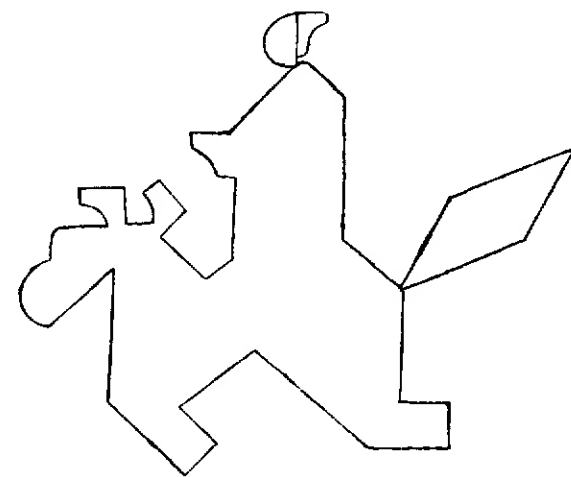
327

鼎三足
兩耳
和五
味之
瑤器也

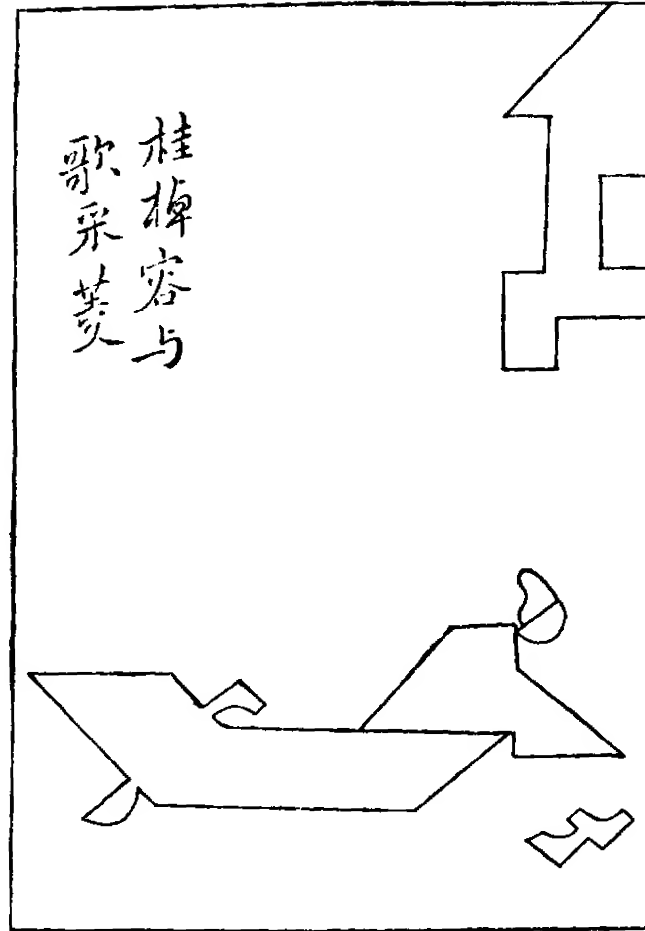


328

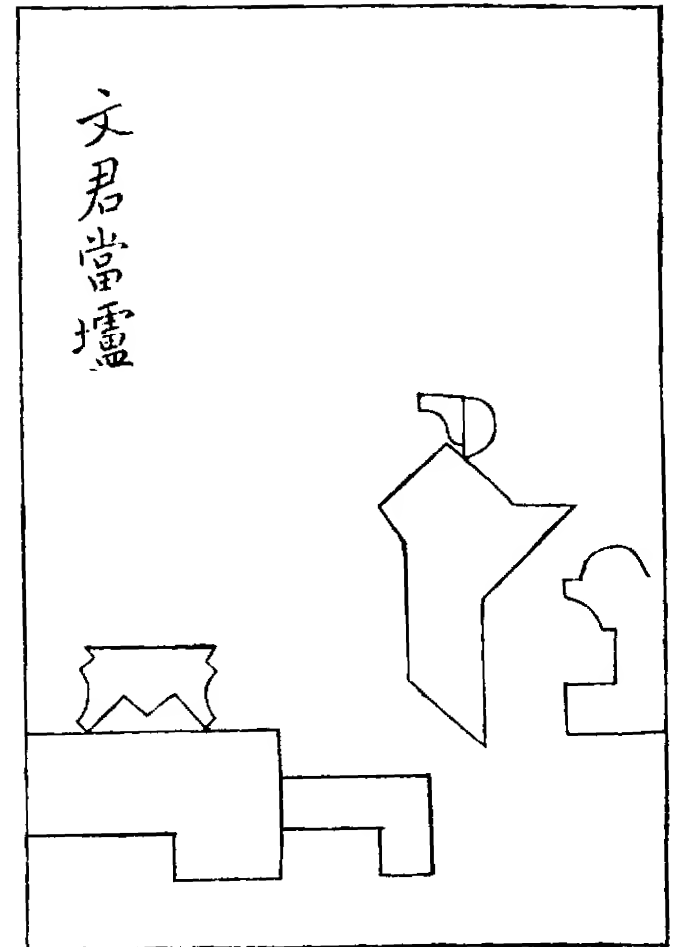
桃花馬上
石榴裙



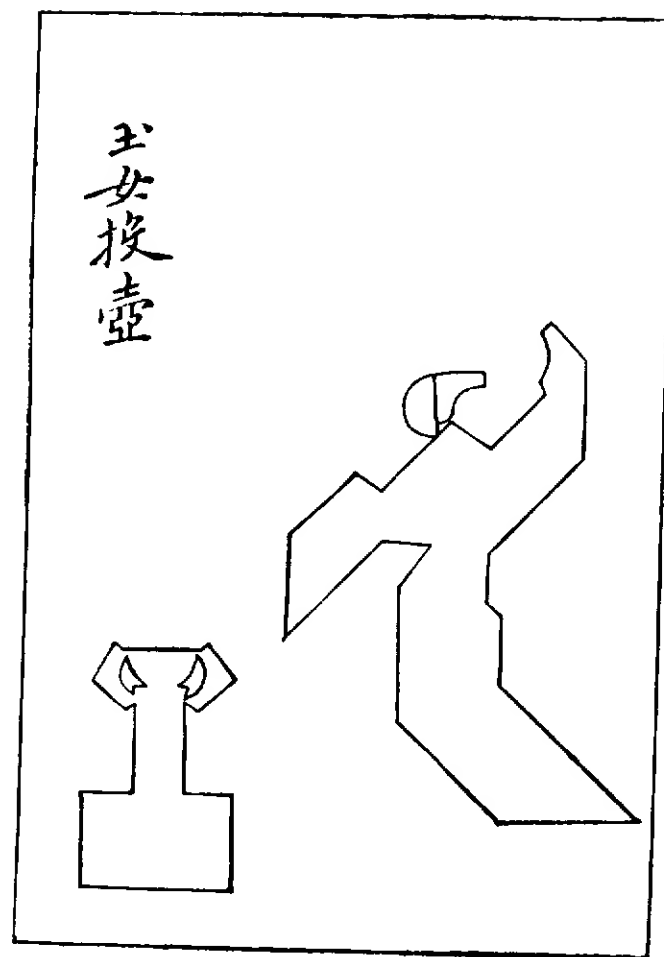
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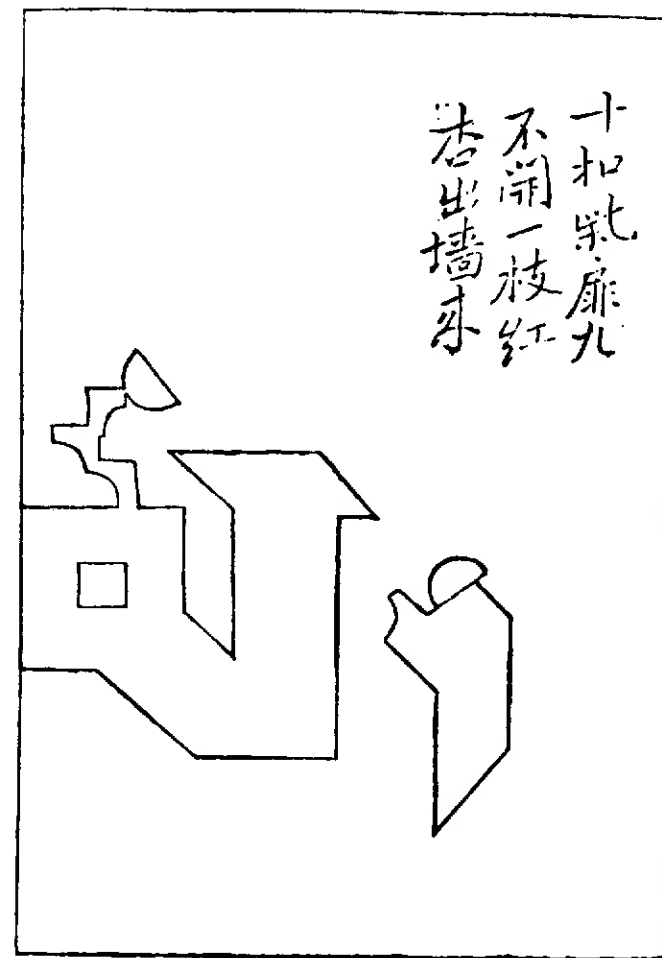
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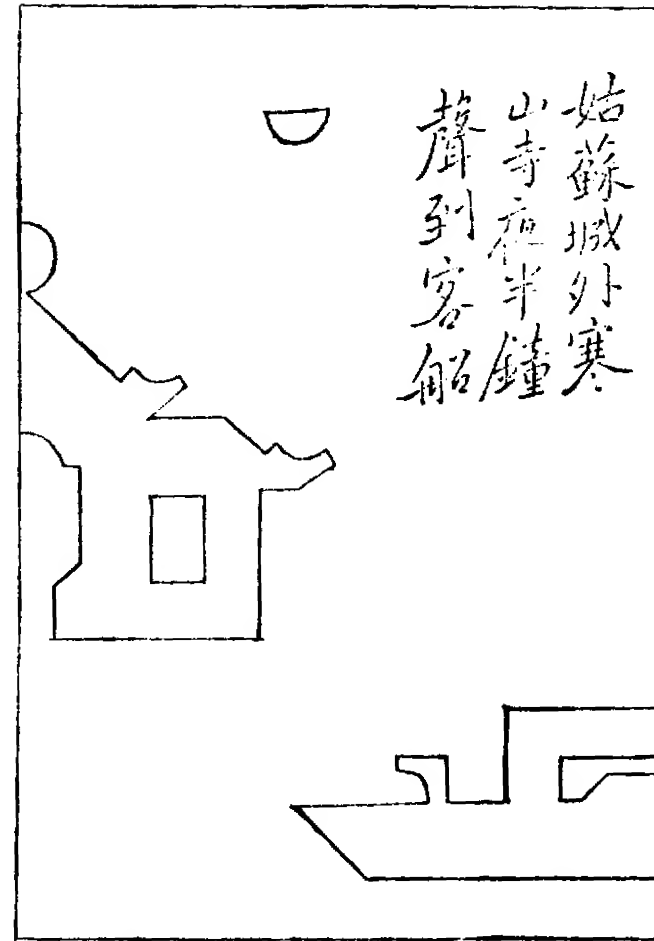
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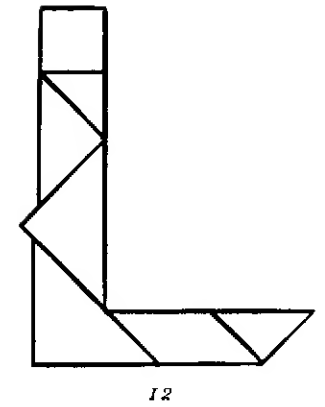
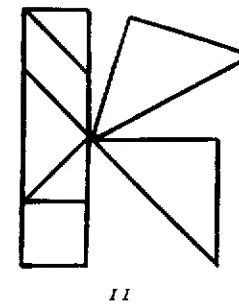
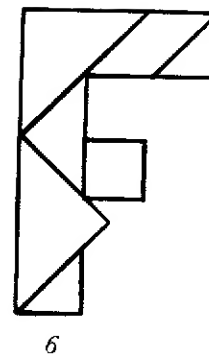
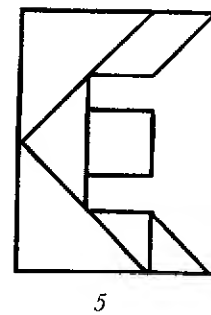
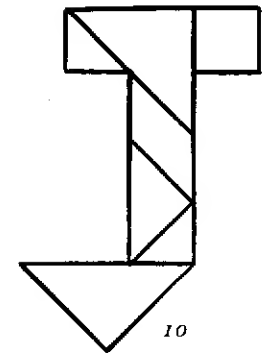
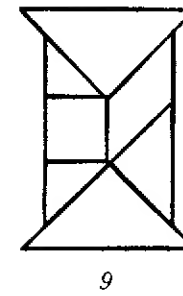
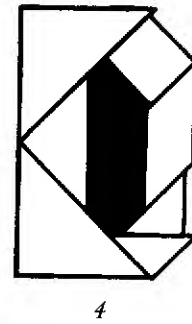
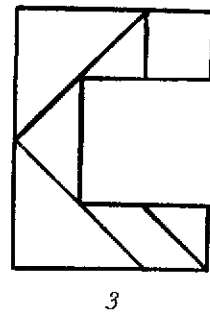
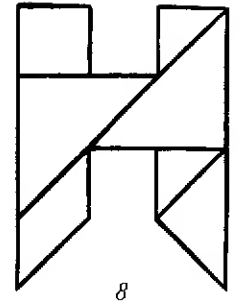
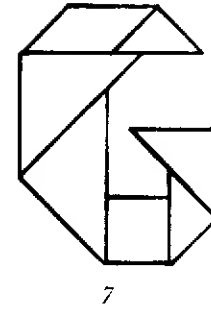
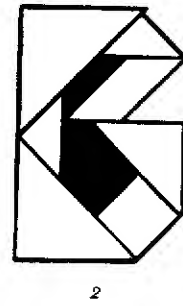
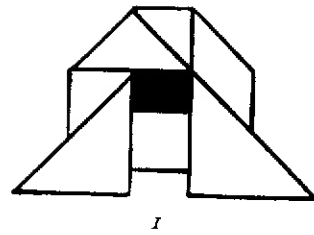
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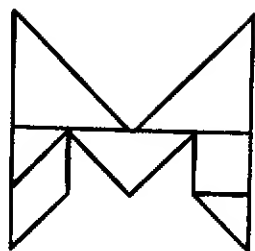


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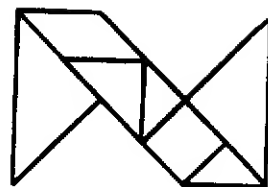


Solutions

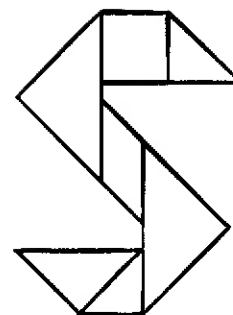




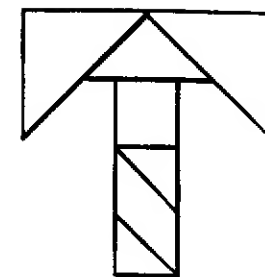
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14



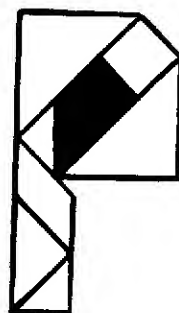
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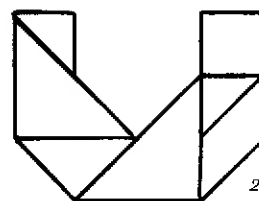
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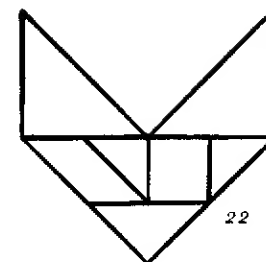
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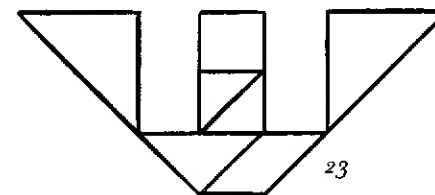
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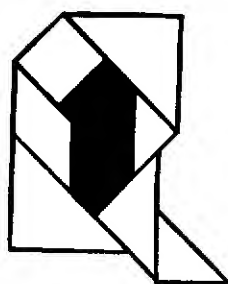
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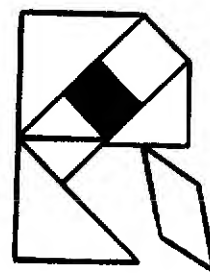
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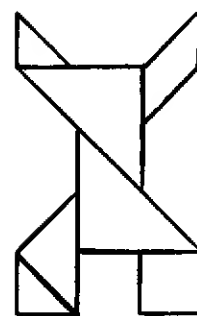
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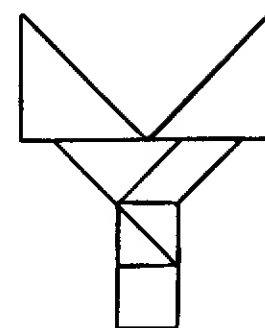
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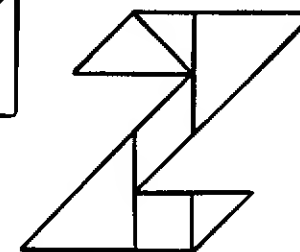
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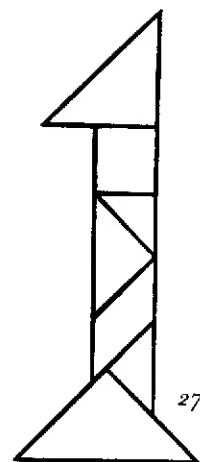
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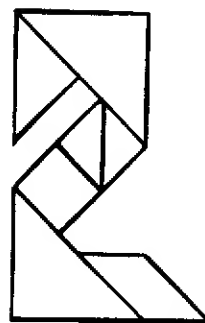
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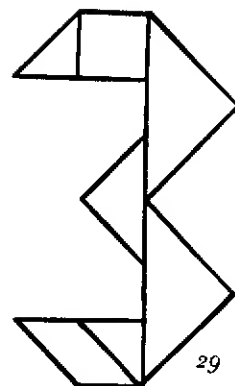
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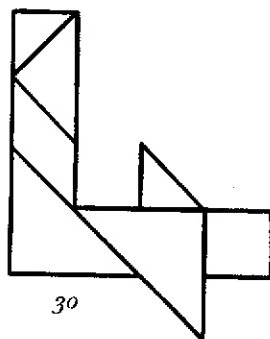
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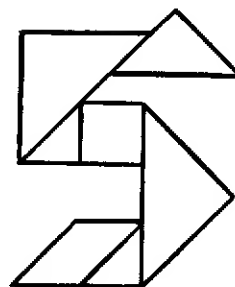
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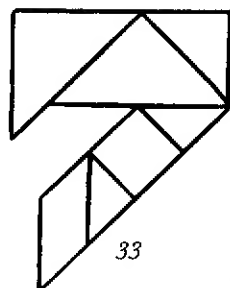
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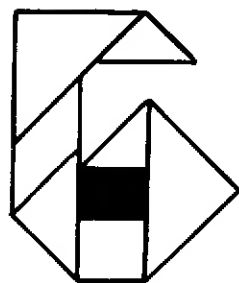
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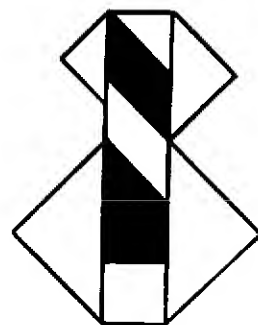
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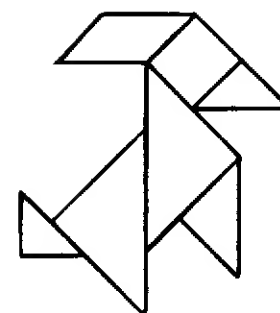


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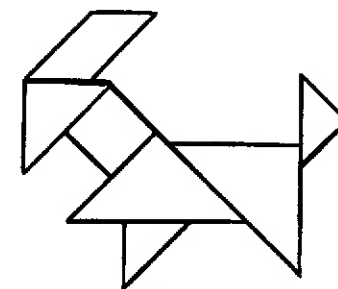
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THE TANGRAM ZOO



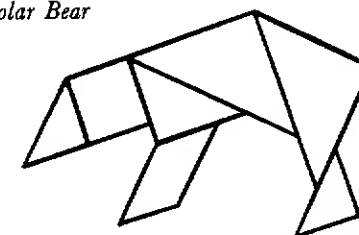
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GOATS

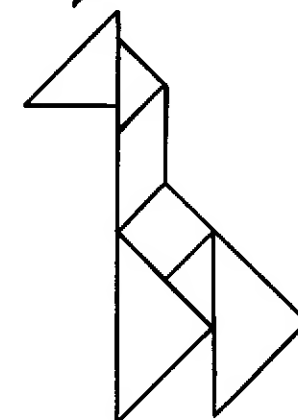


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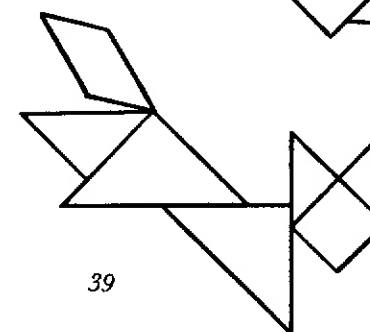
Polar Bear



37

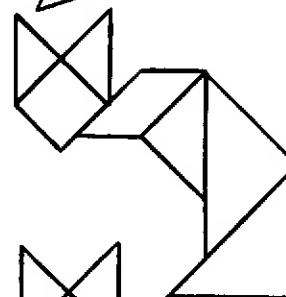


38 Giraffe

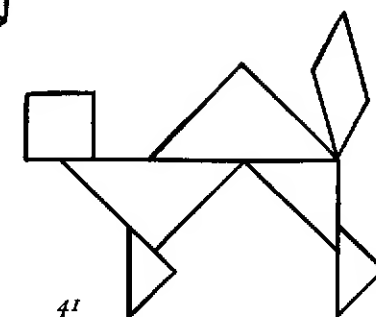


39

CATS

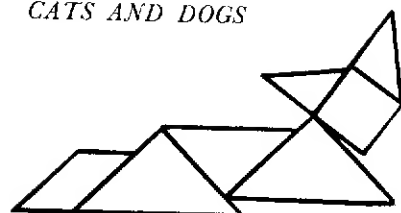


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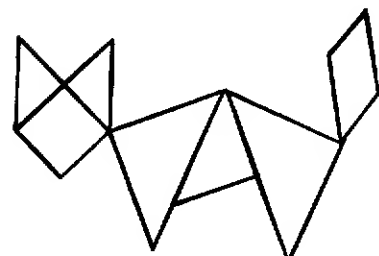


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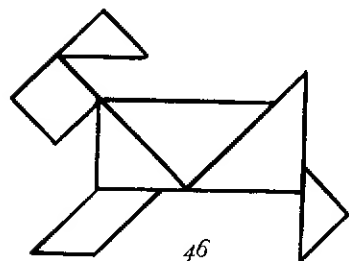
CATS AND DOGS



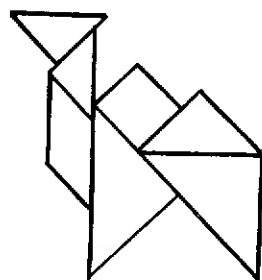
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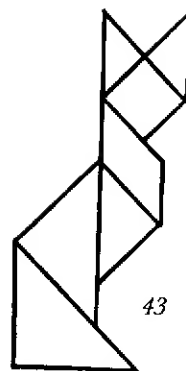
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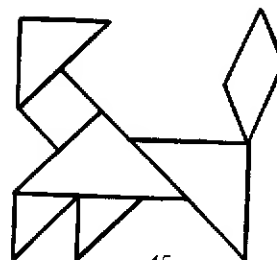
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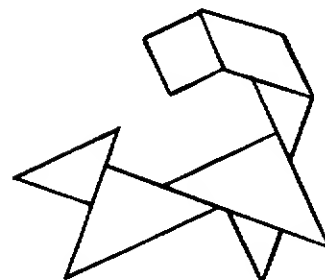
47 Camel



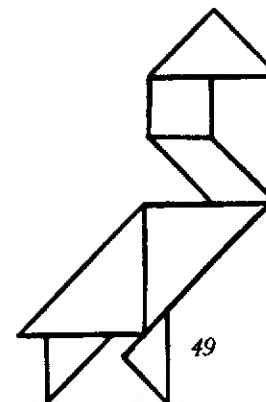
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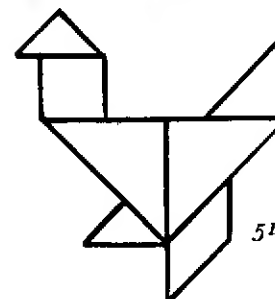
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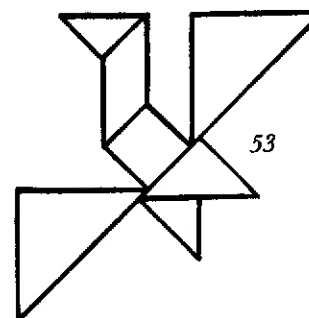
48 Squirrel



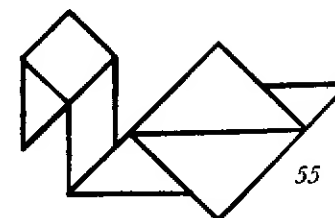
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51

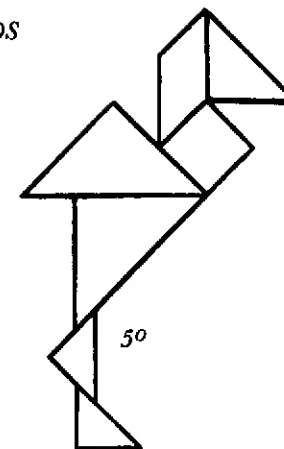


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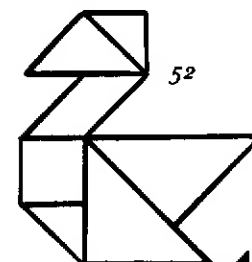


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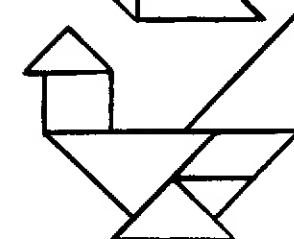
BIRDS



50

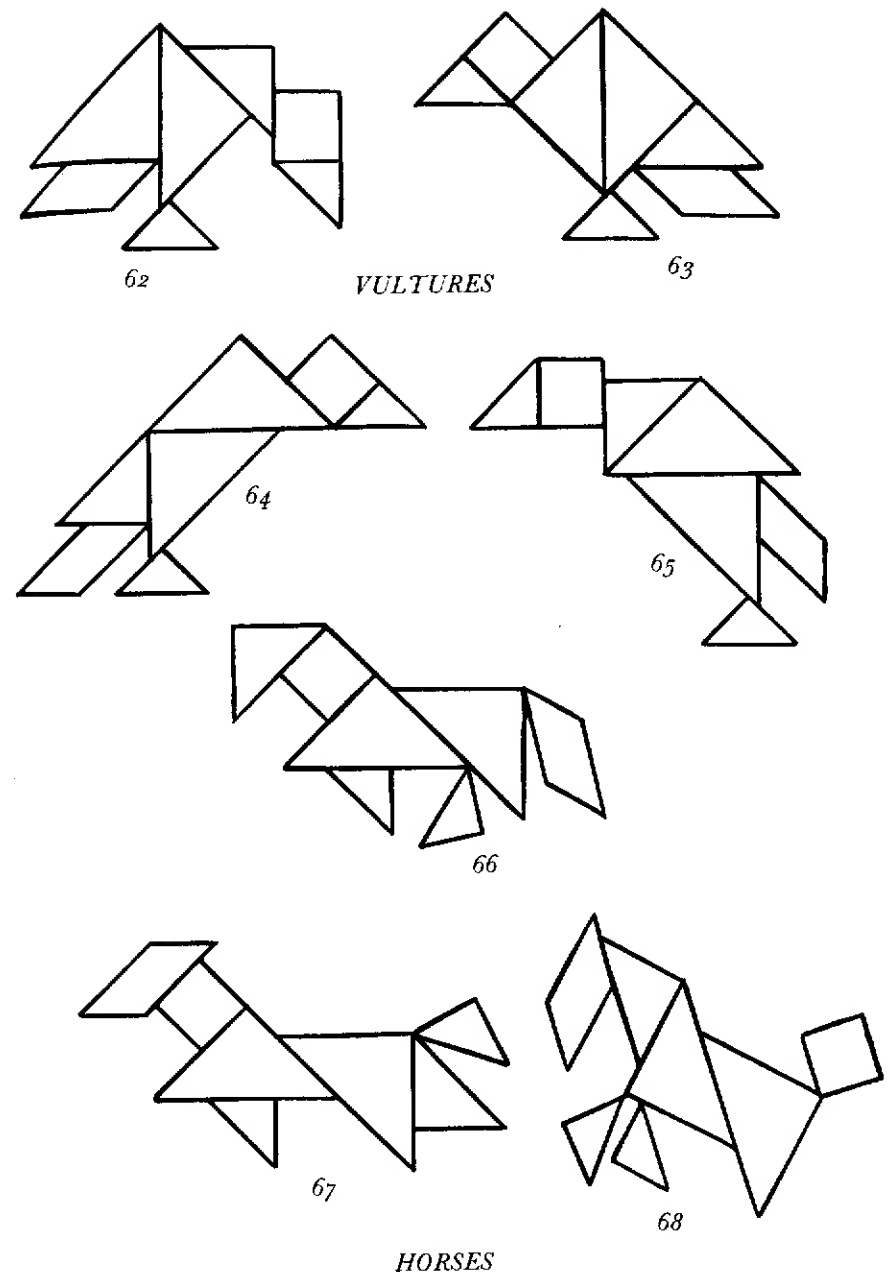
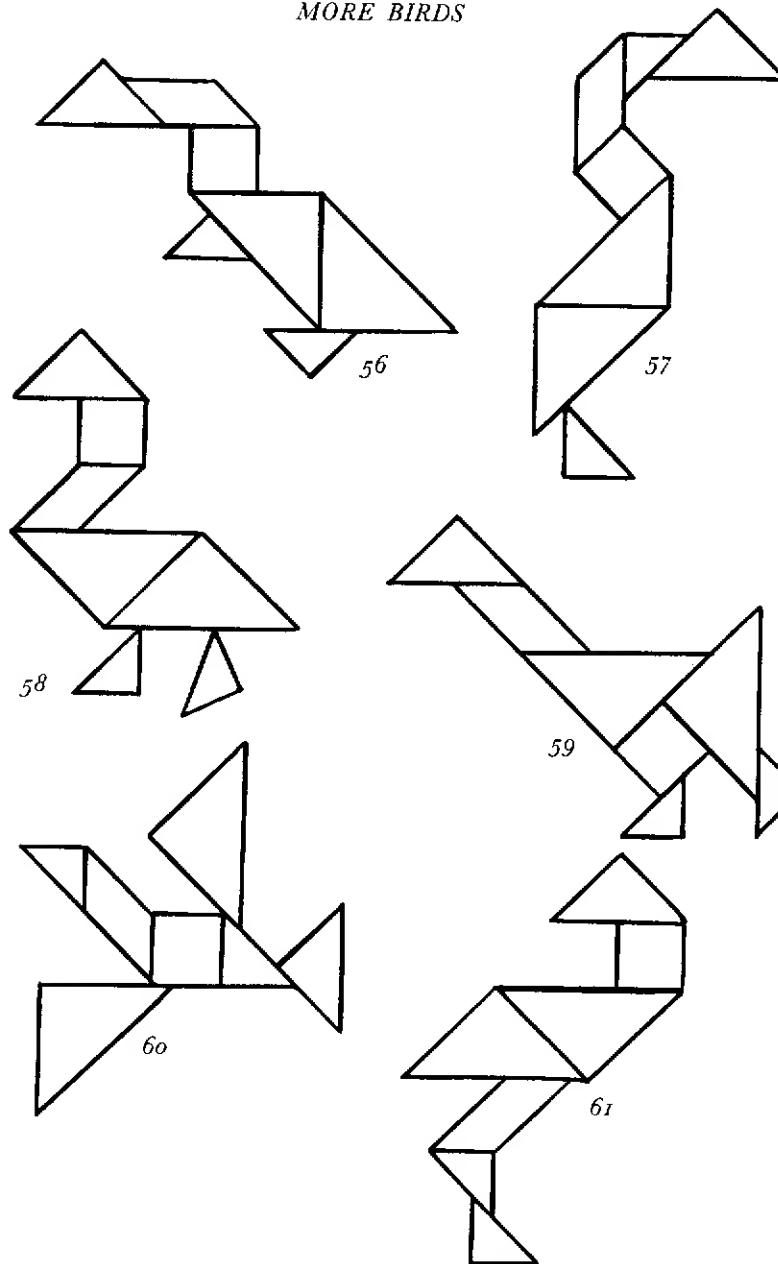


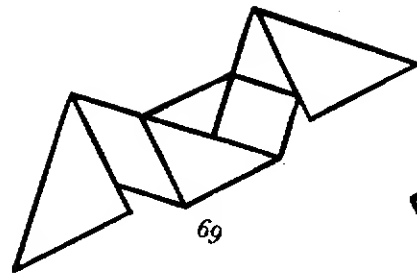
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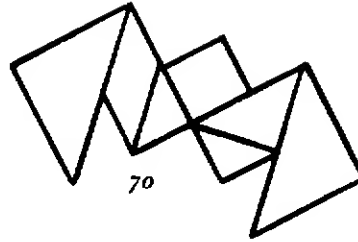
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MORE BIRDS

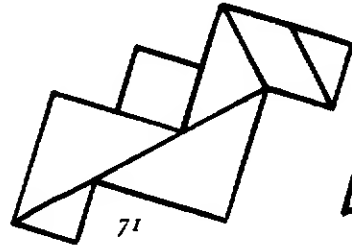




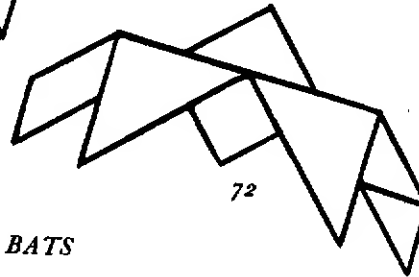
69



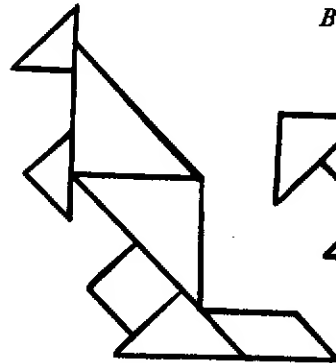
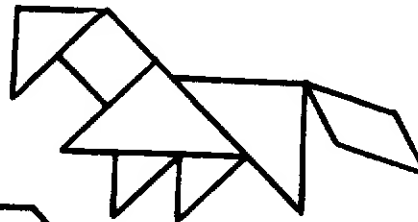
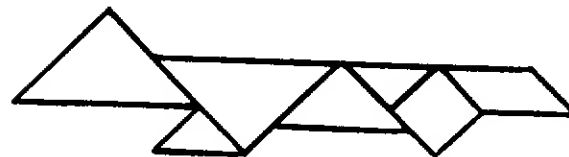
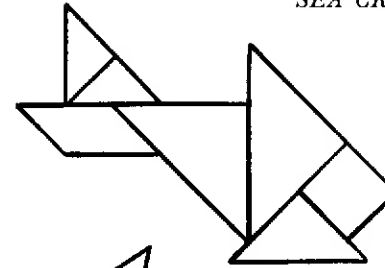
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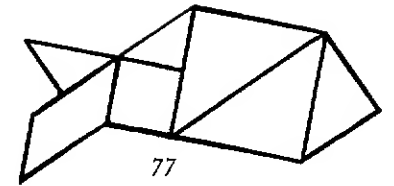
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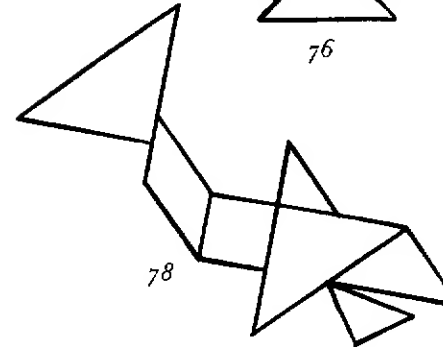
72

BATS73 *Kangaroo*74 *Horse*75 *Crocodile**SEA CREATURES*

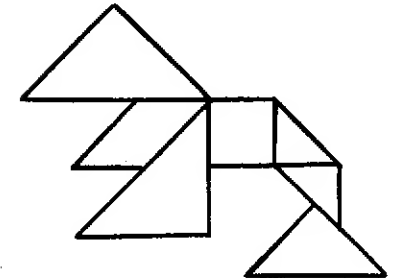
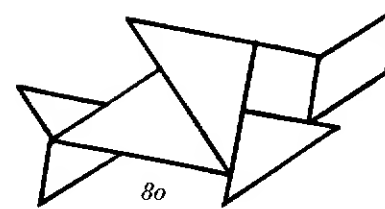
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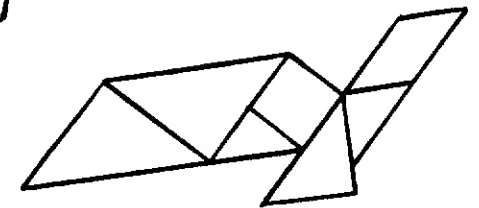
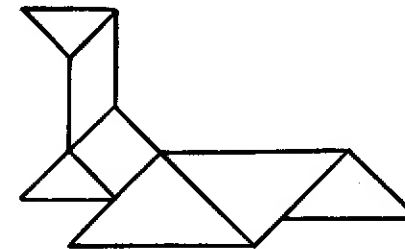
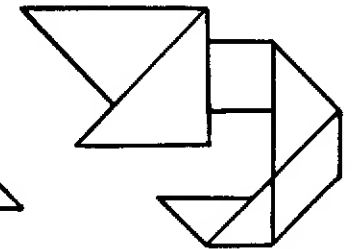
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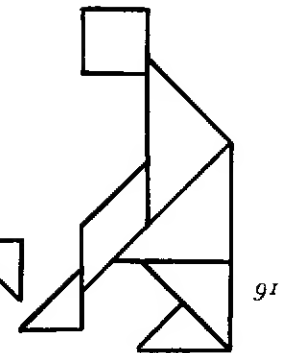
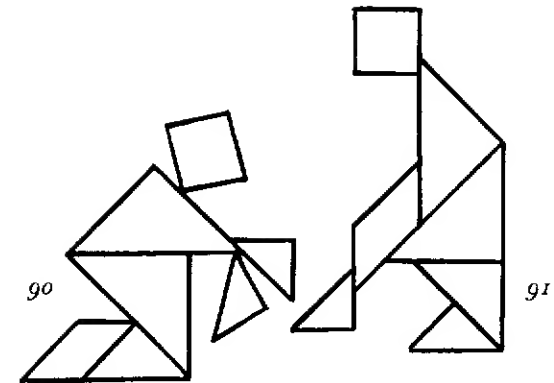
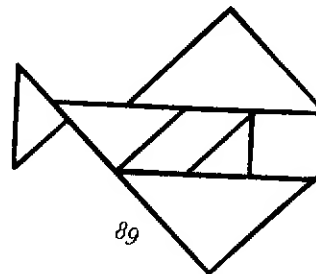
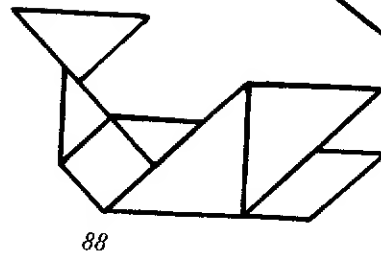
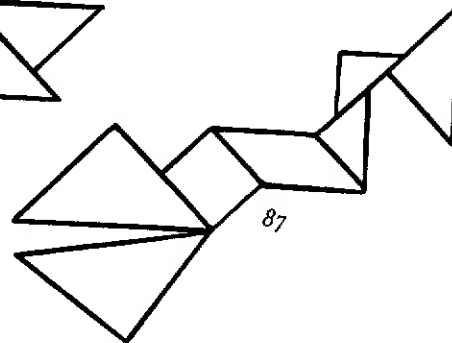
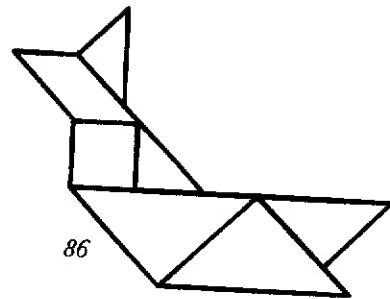
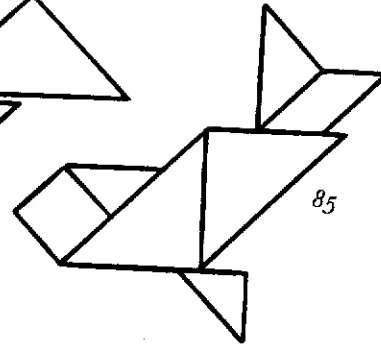
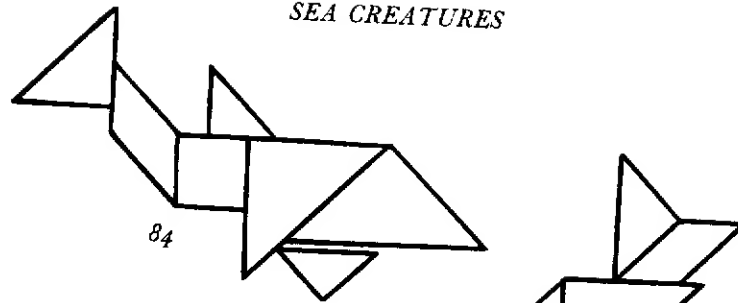
78

79 *Lobster*

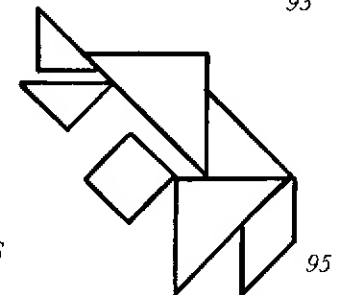
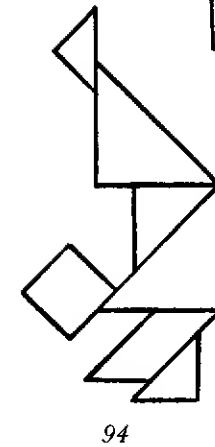
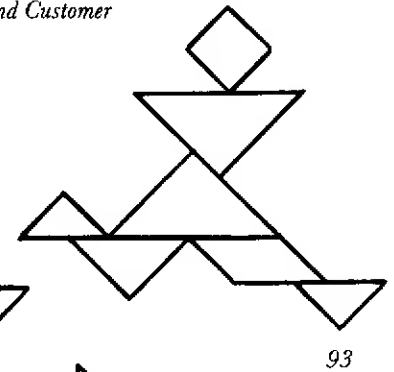
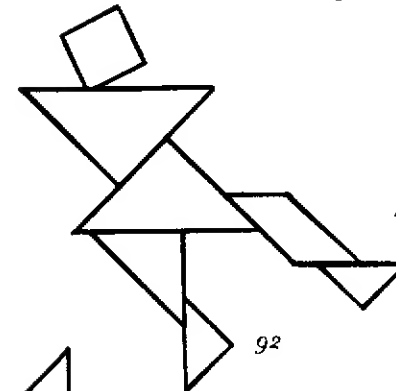
80

81 *Turtle*82 *Seal*83 *Shrimp*

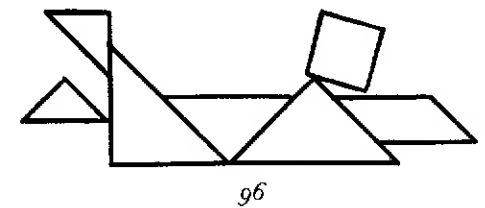
SEA CREATURES

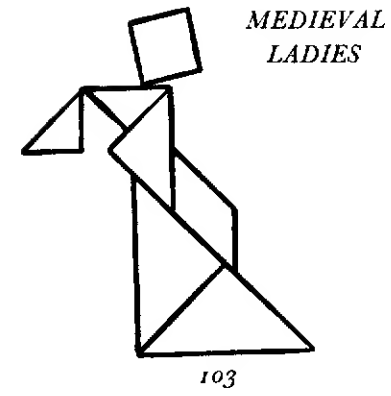
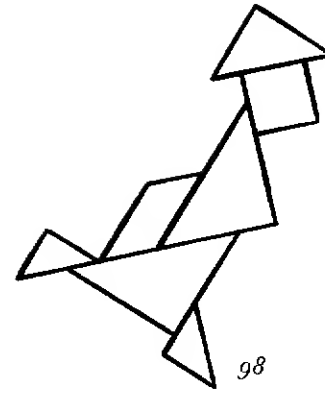
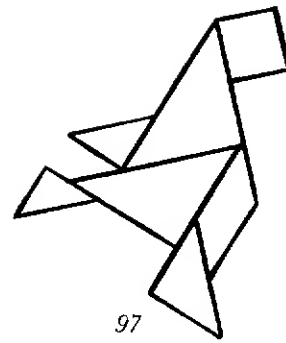


Shoeshine Boy and Customer

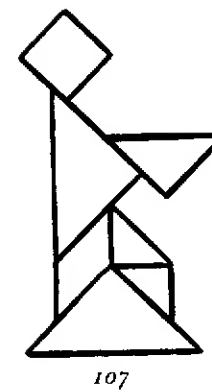
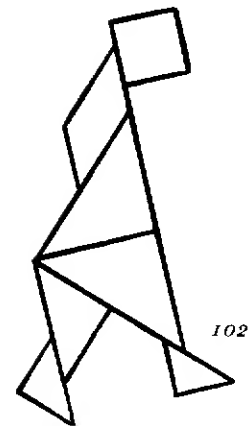
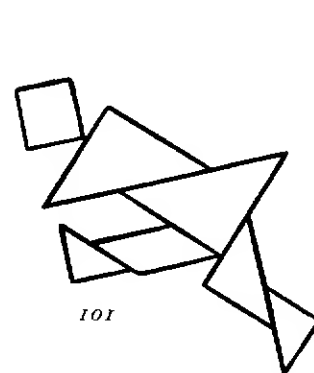
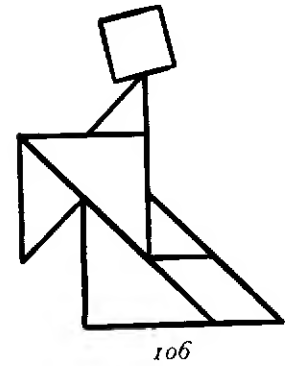
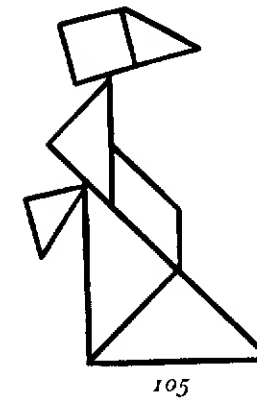
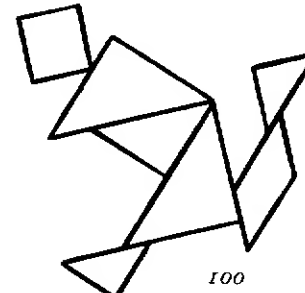
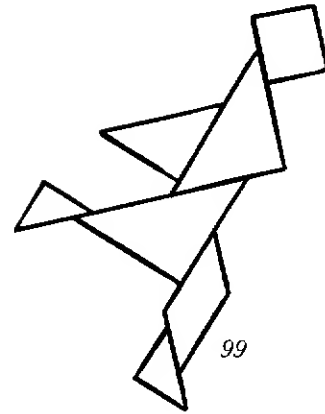
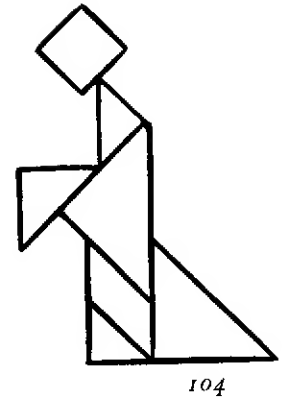


ACROBATS

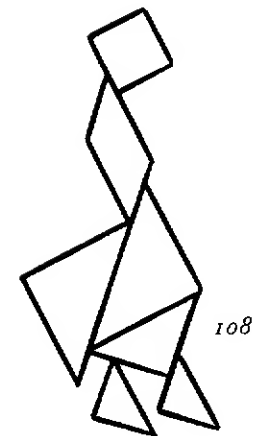




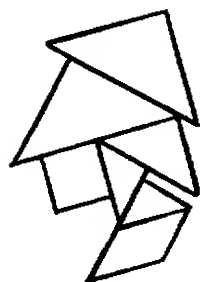
*MEDIEVAL
LADIES*



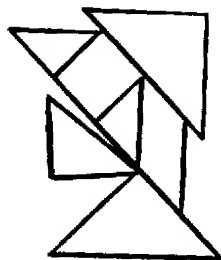
AND SERVANTS



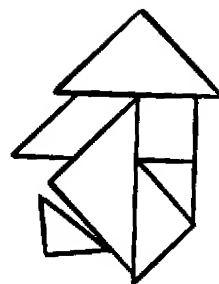
SAM LOYD'S PORTRAIT GALLERY



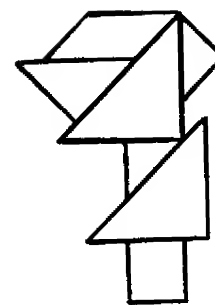
109



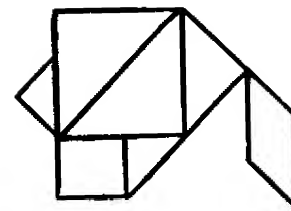
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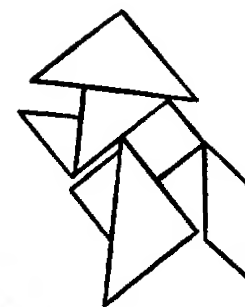
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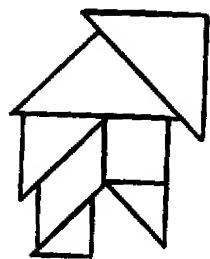
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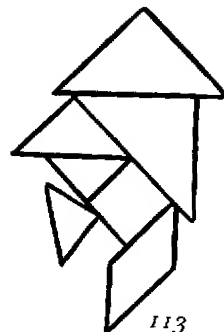
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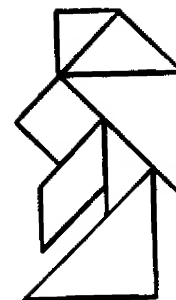
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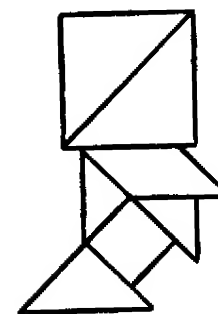
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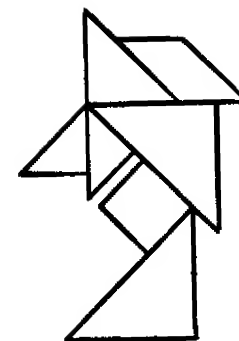
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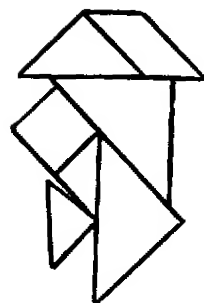
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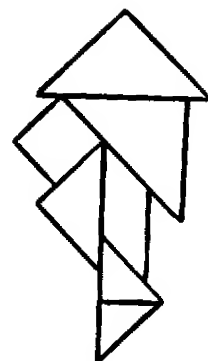
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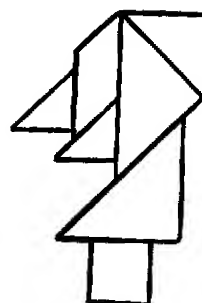
122



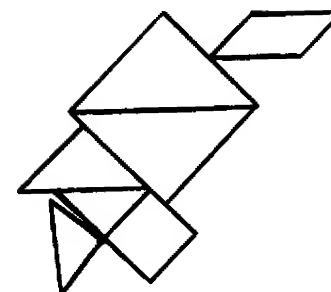
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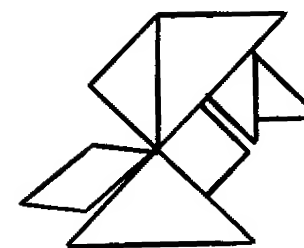
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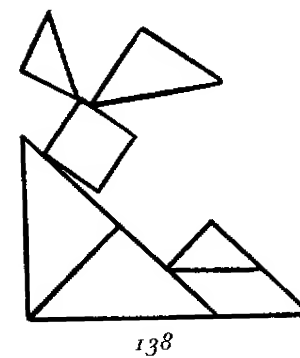
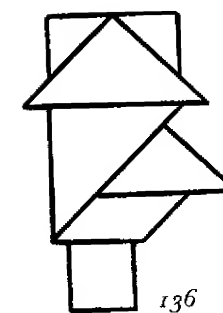
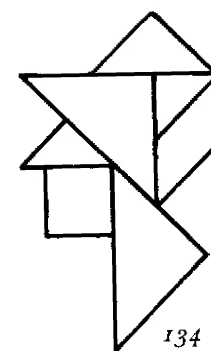
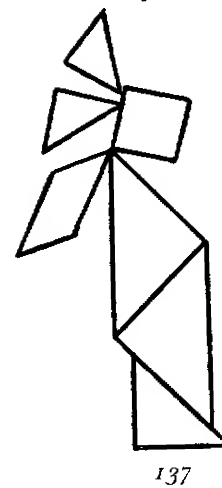
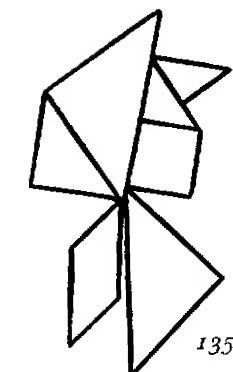
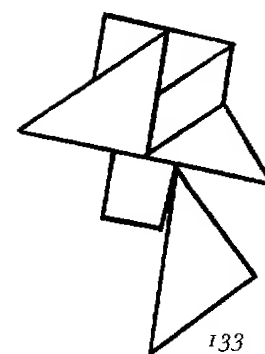
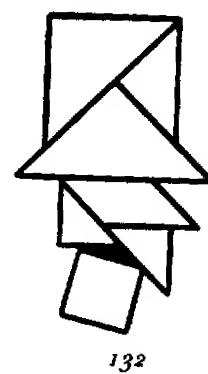
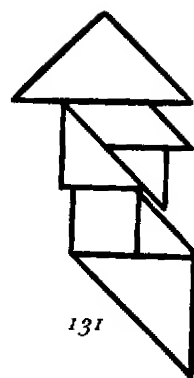
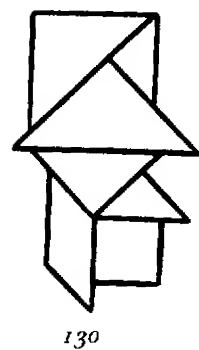
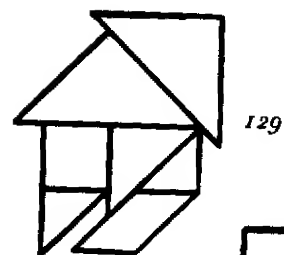
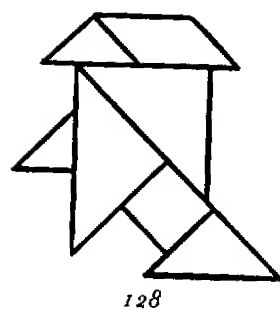
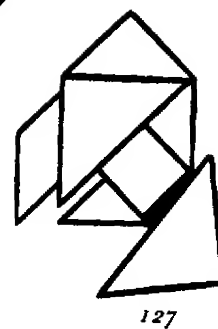
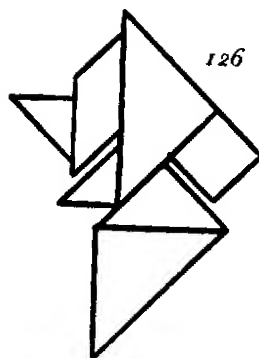
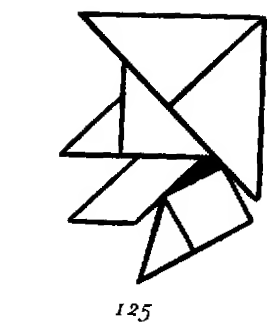
116



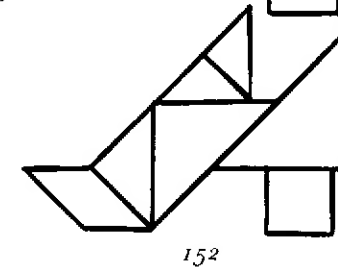
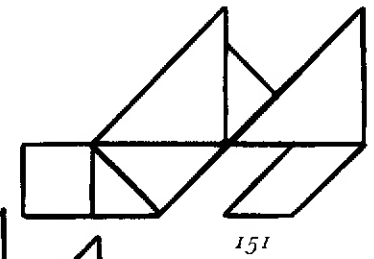
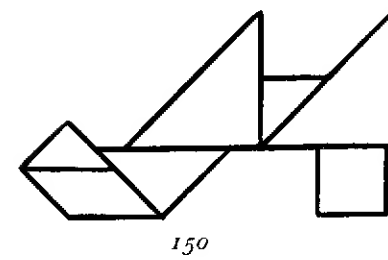
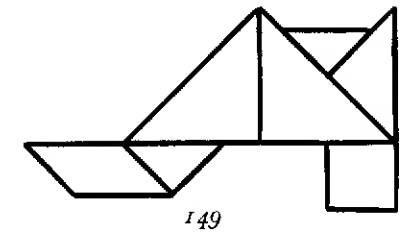
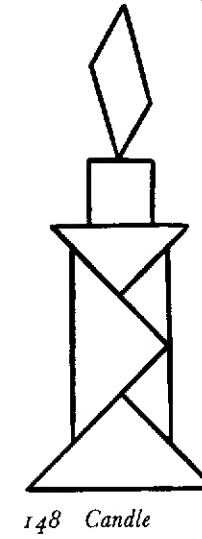
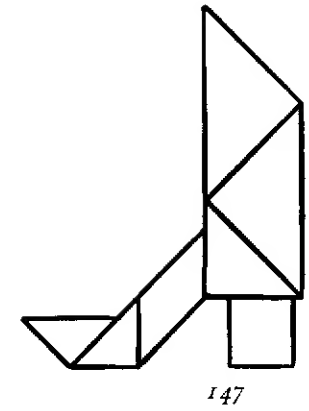
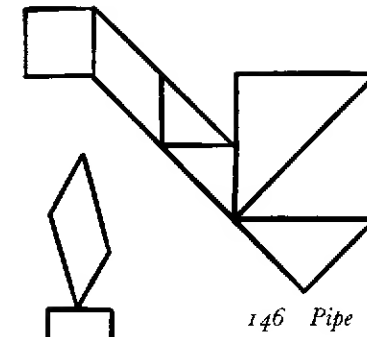
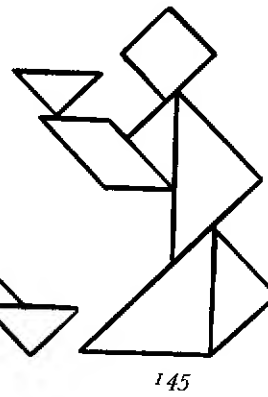
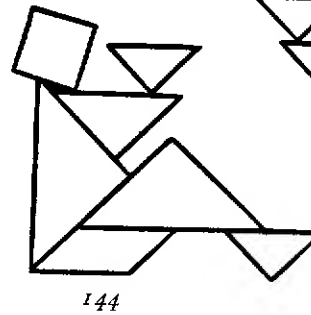
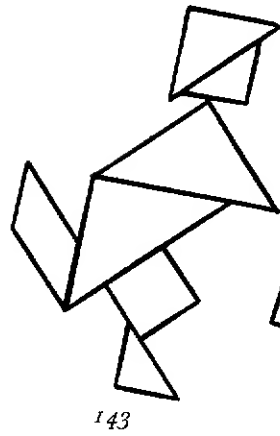
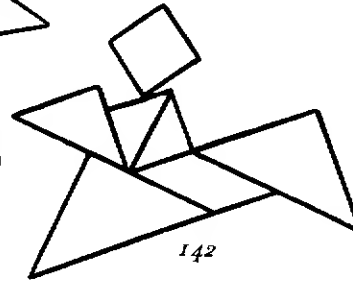
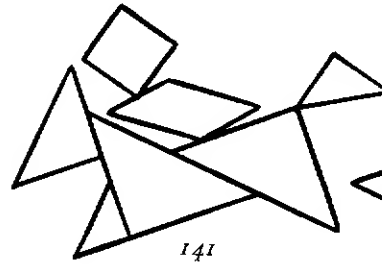
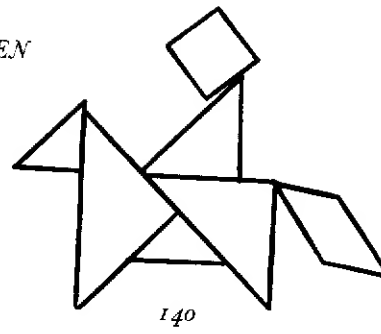
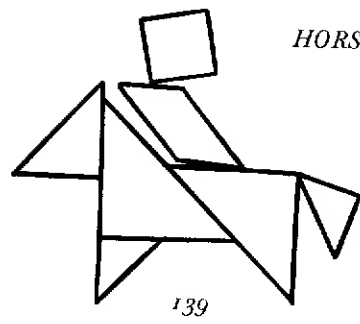
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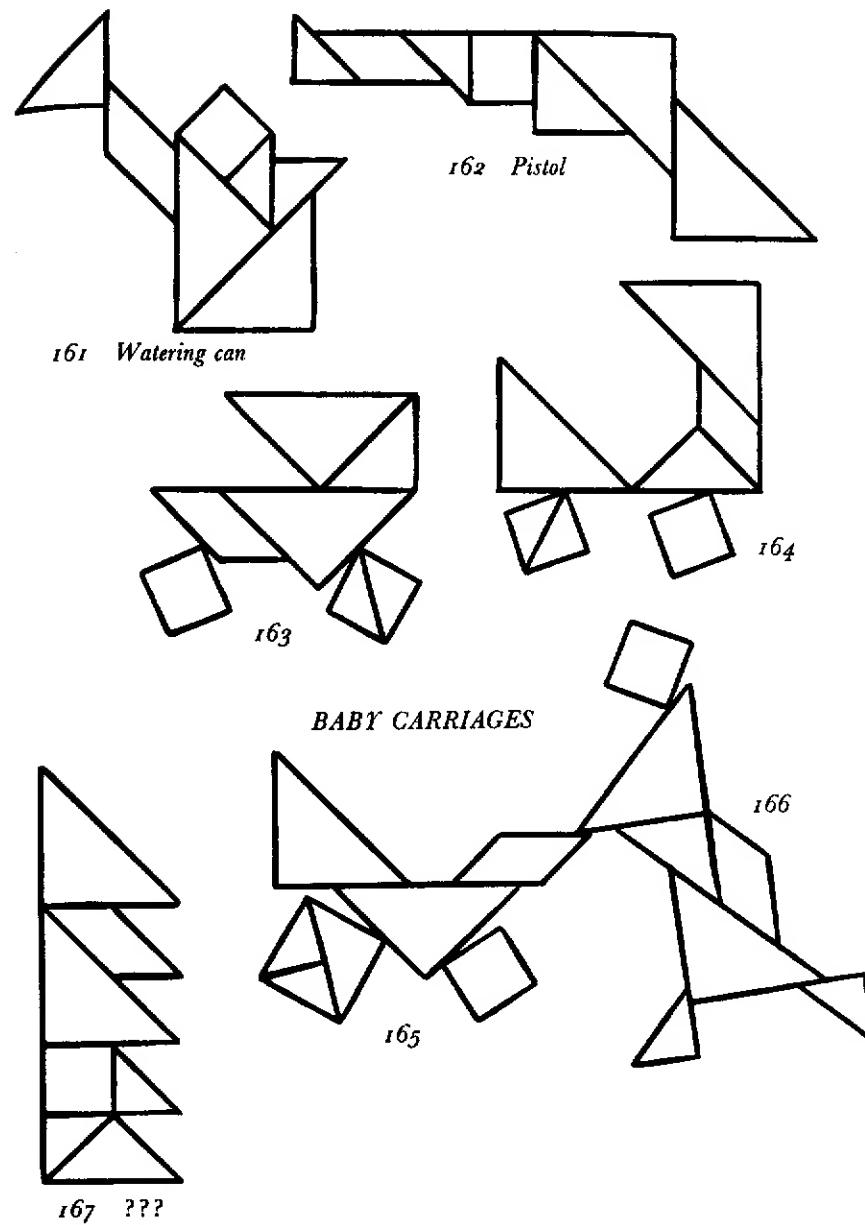
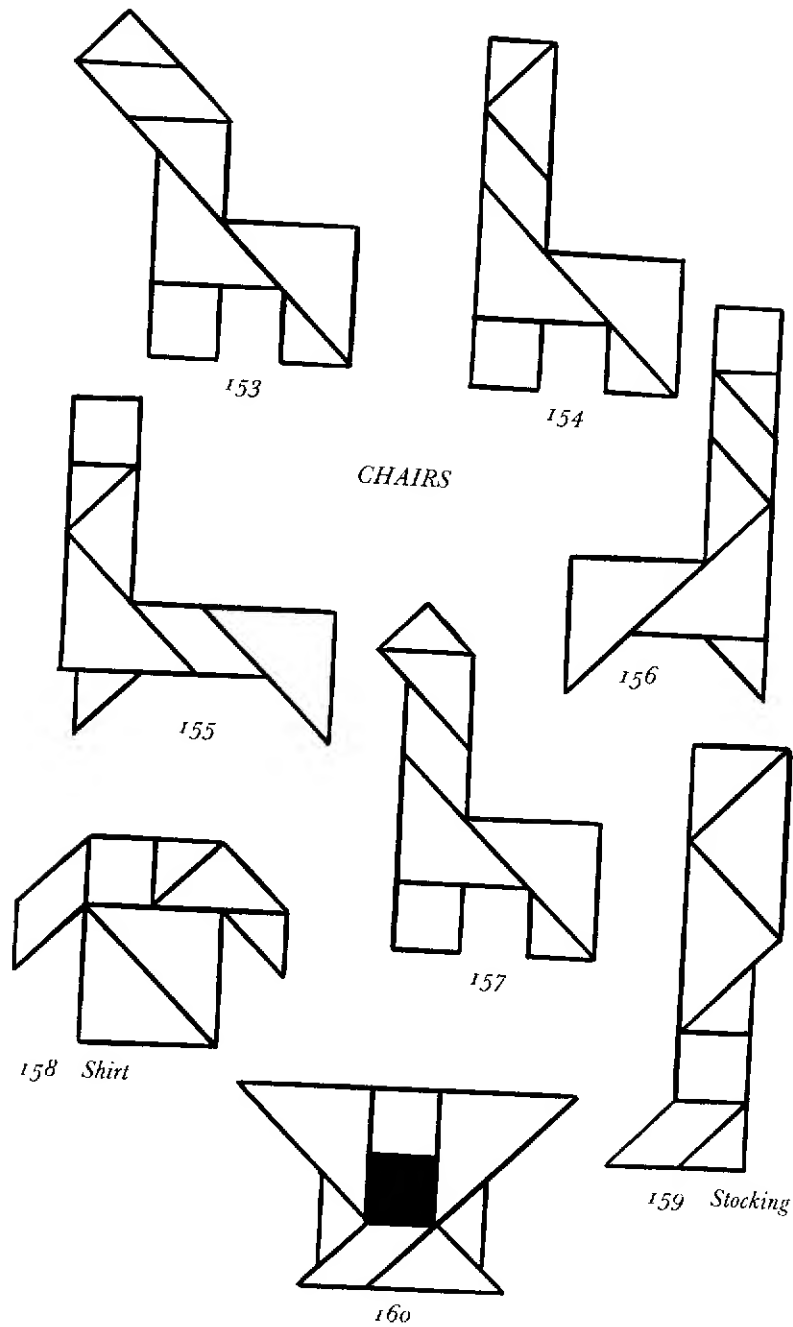
124



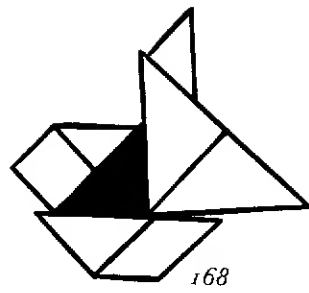
HORSEMEN



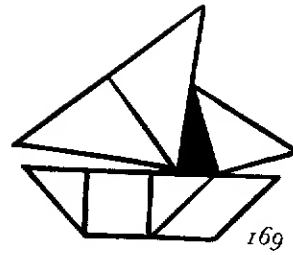
SHOES



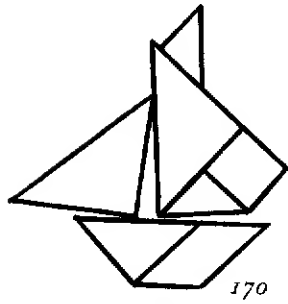
BOATS



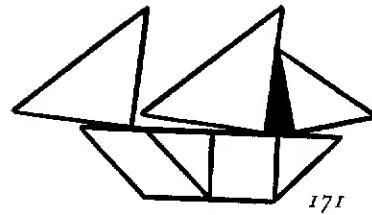
168



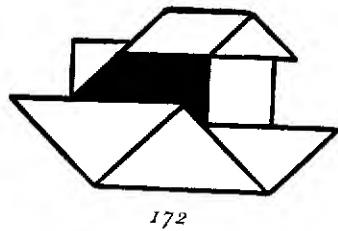
169



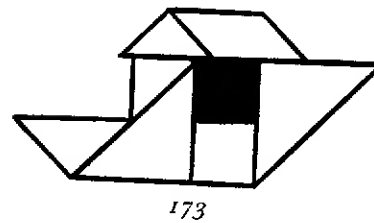
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171

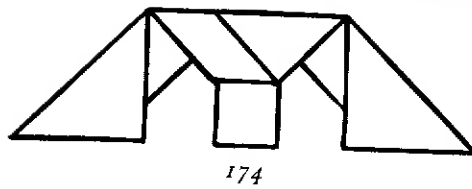


172

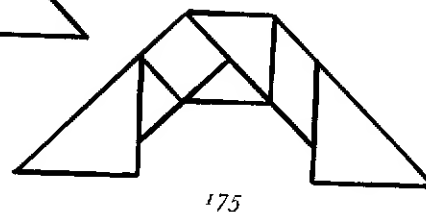


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AND BRIDGES

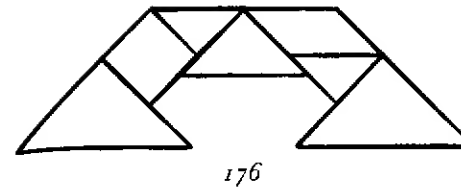


174

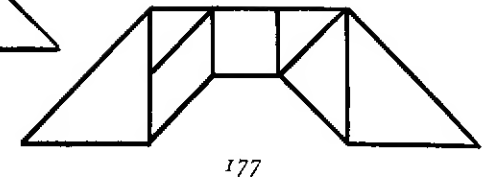


175

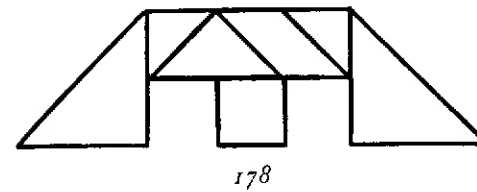
BRIDGES



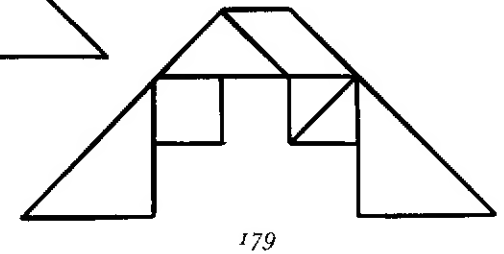
176



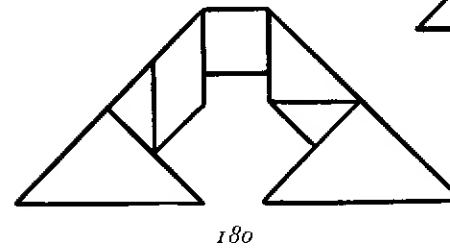
177



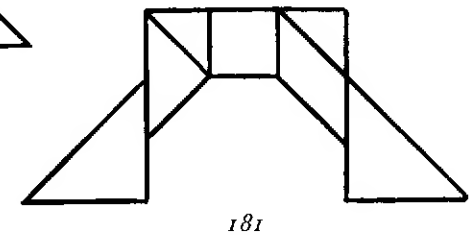
178



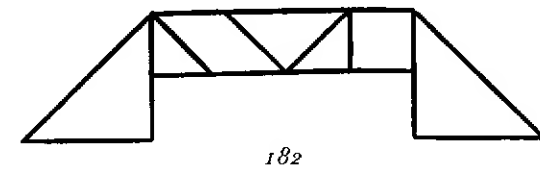
179



180

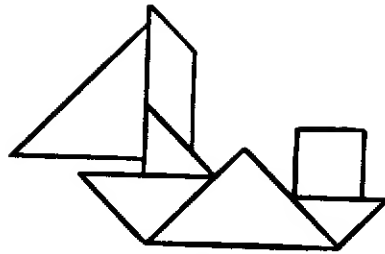


181

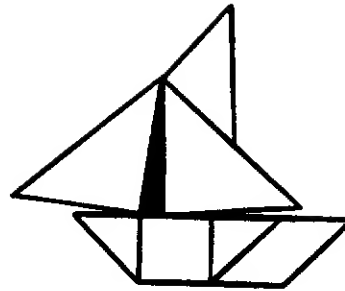


182

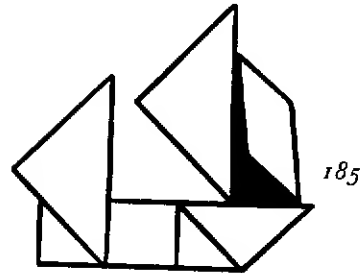
BOATS



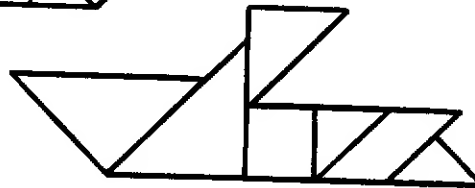
183



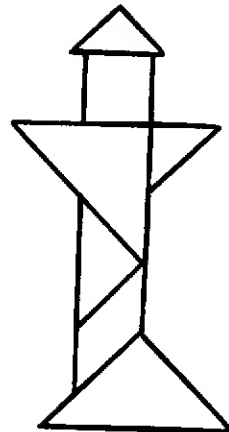
184



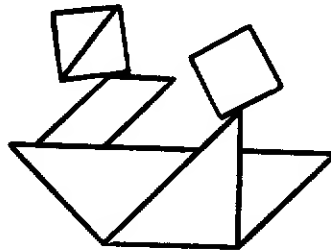
185



186

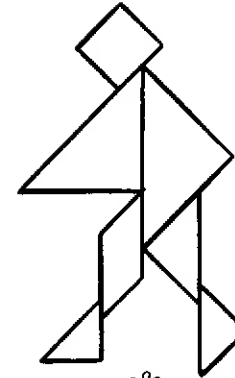


187 & 188 Rowing to the lighthouse

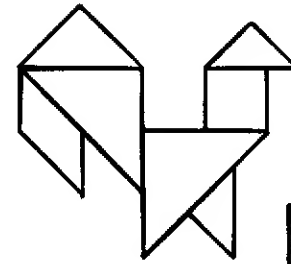


THE HOUSE THAT JACK BUILT

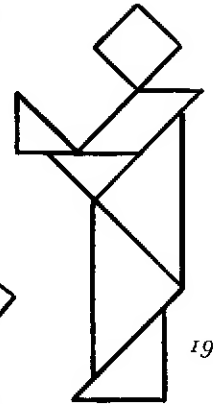
Illustrated by Sam Loyd



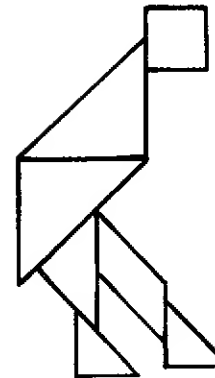
189



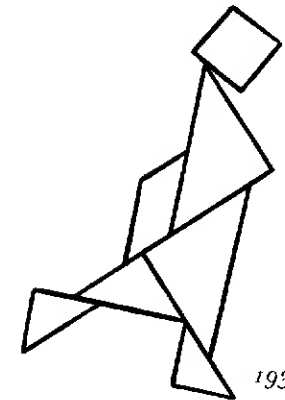
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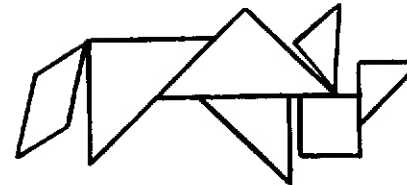
191



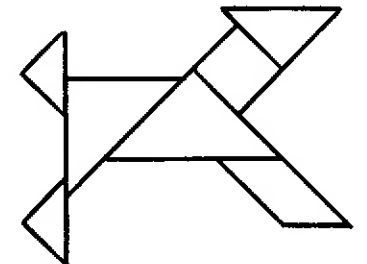
192



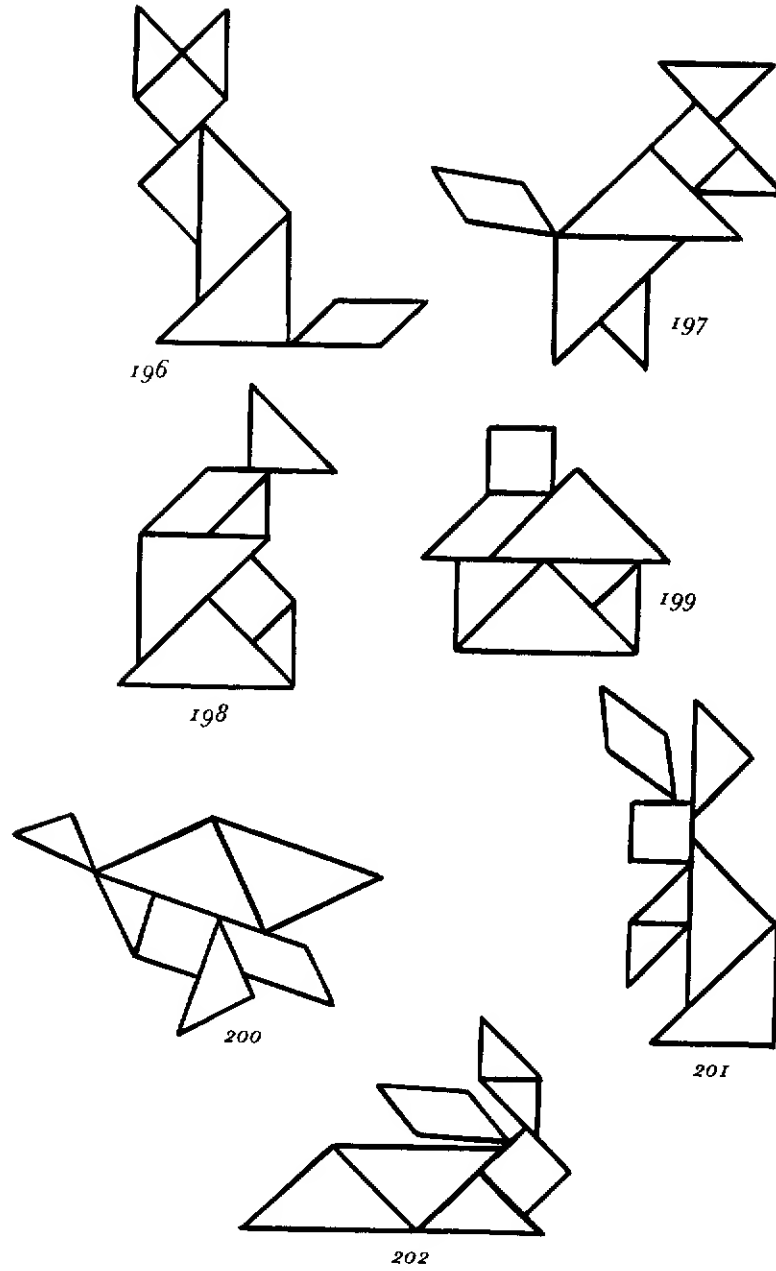
193



194

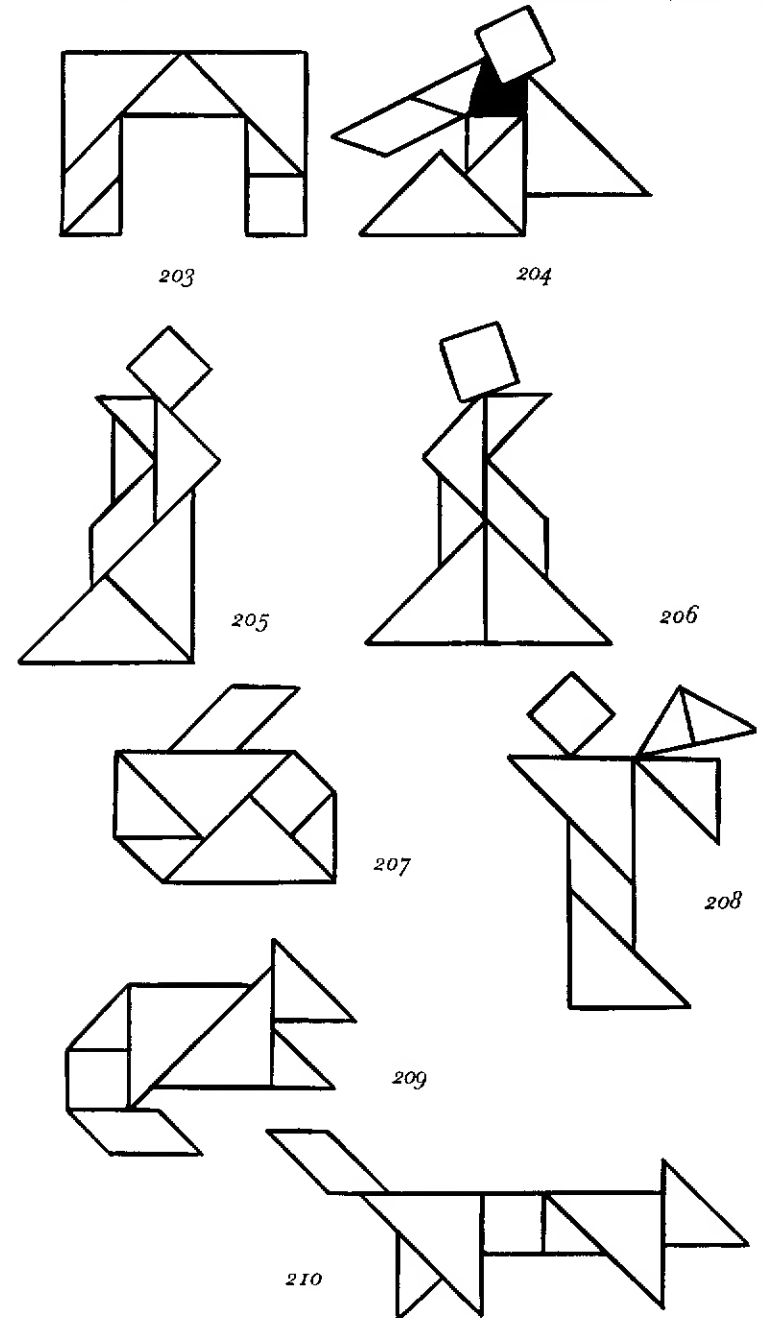


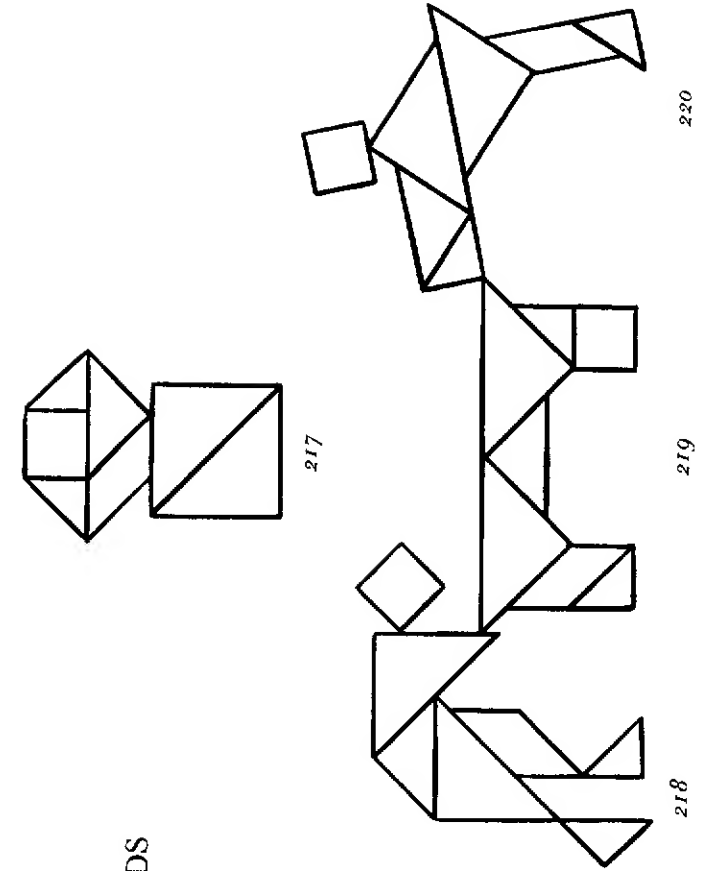
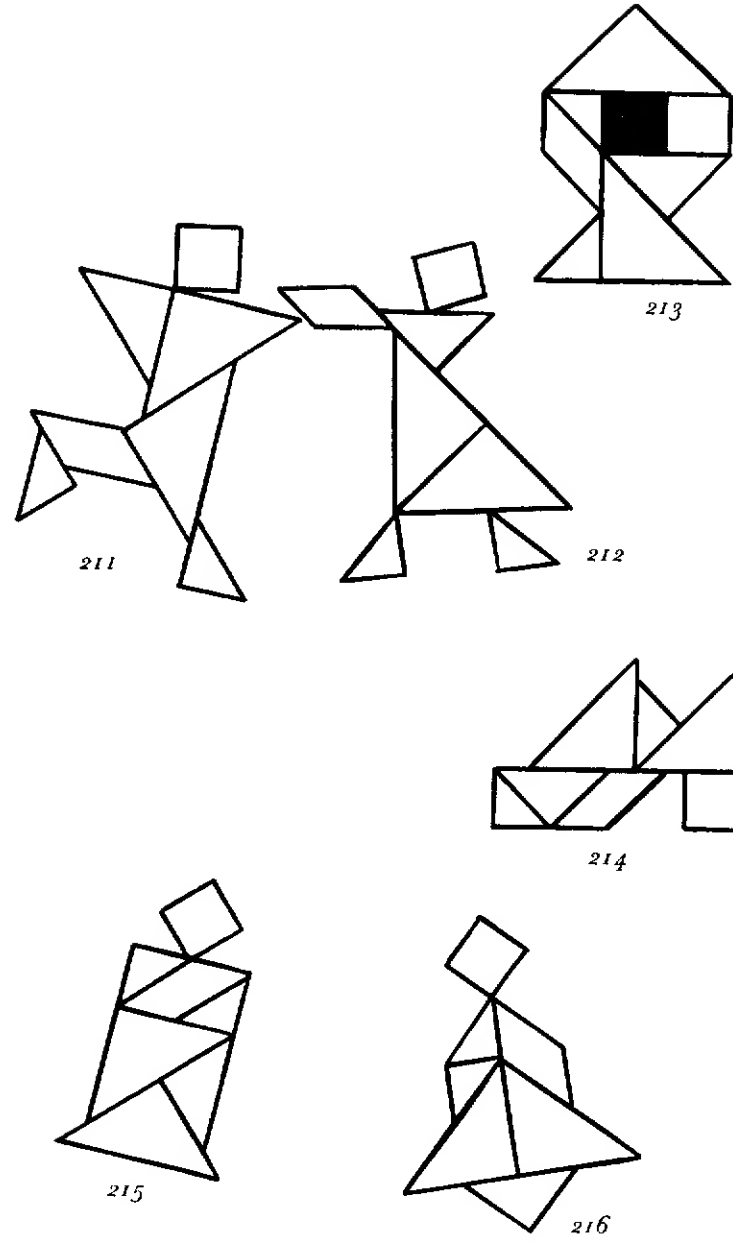
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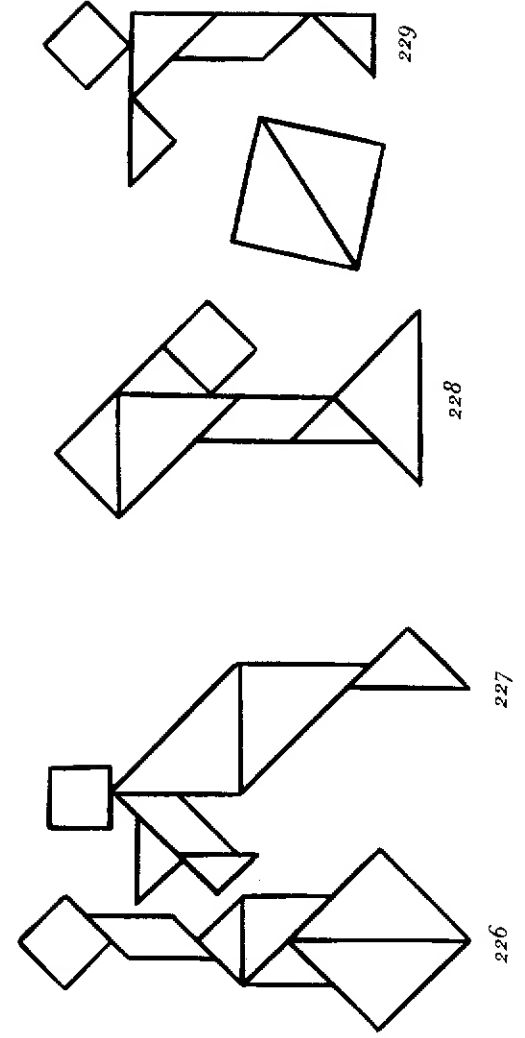
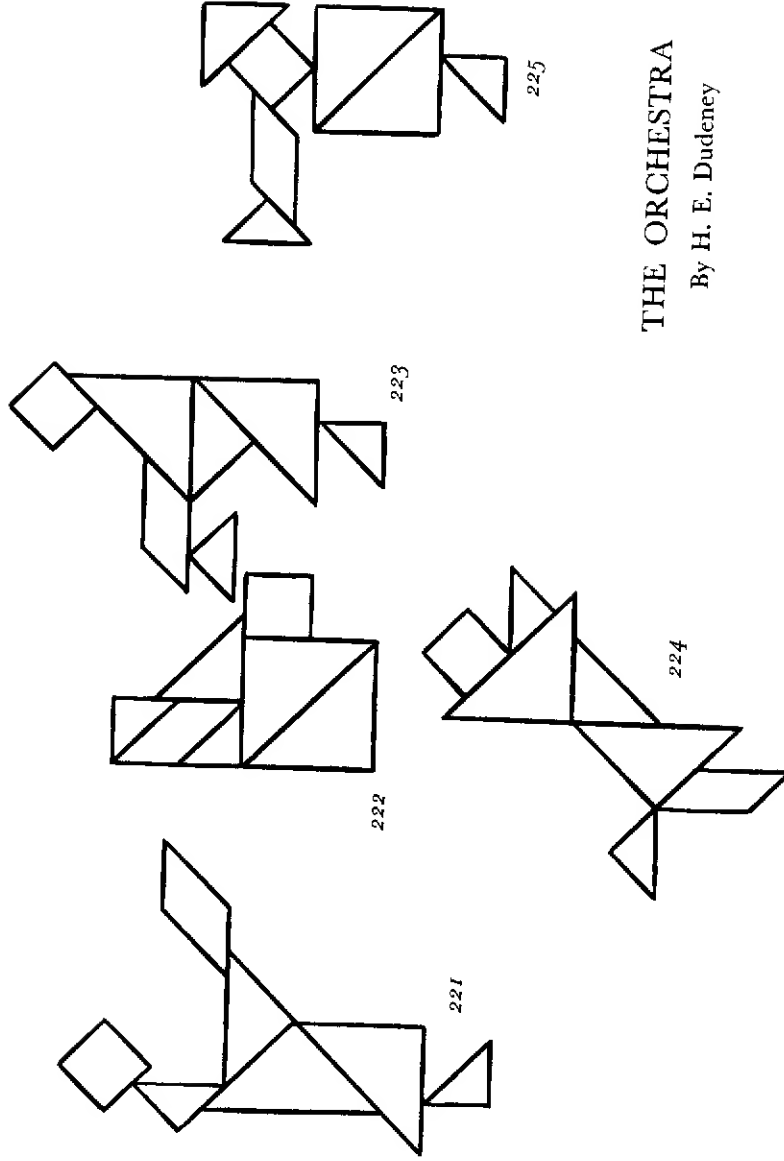
THE STORY OF CINDERELLA

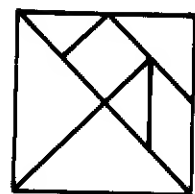
Illustrated by Sam Loyd



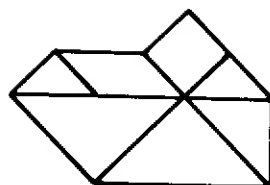


A GAME OF BILLIARDS
By H. E. Dudeney

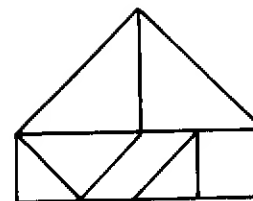




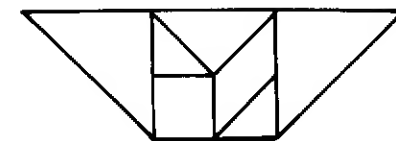
230



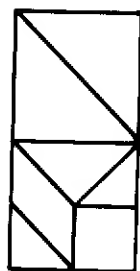
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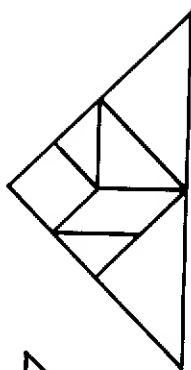
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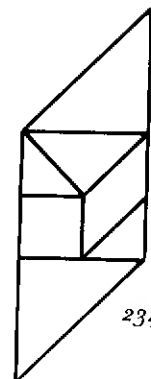
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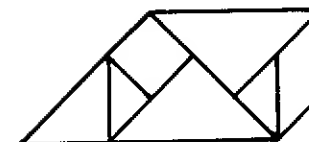
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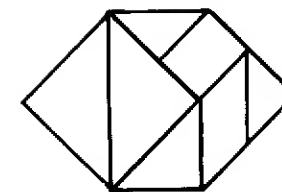
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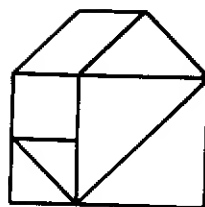
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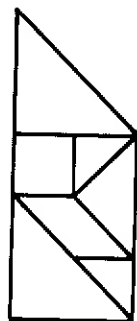
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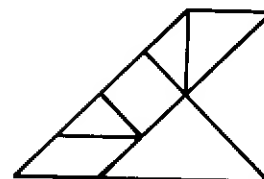
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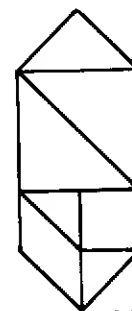
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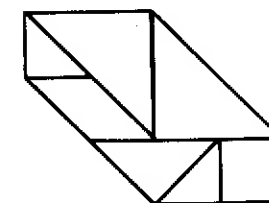
236



237

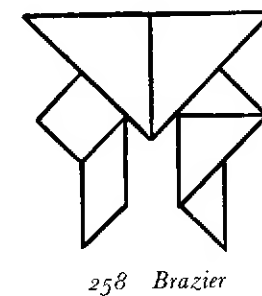
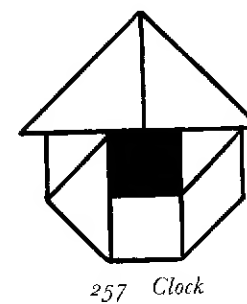
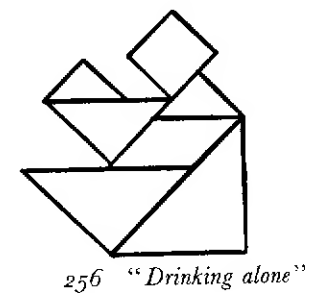
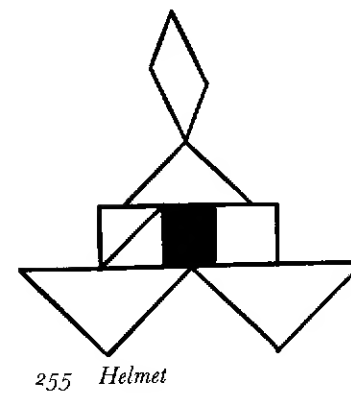
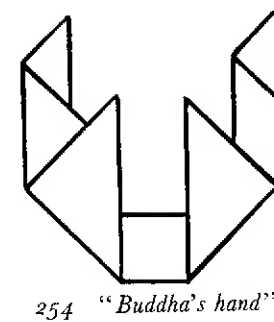
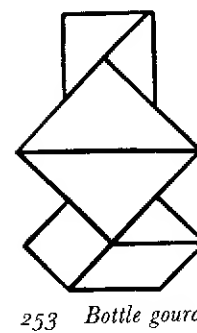
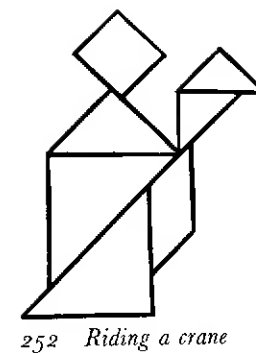
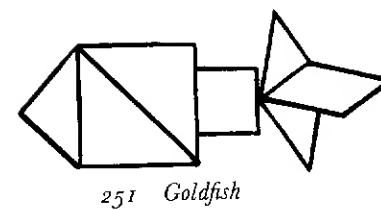
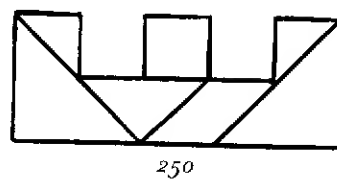
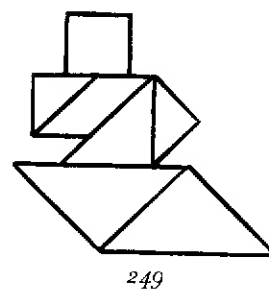
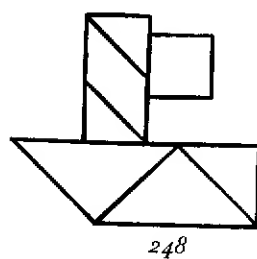
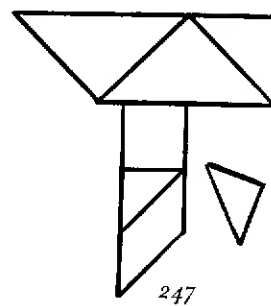
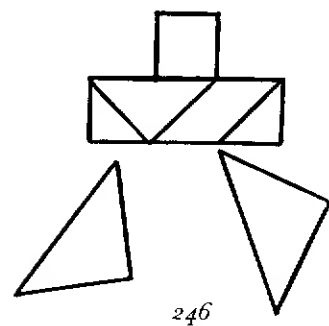
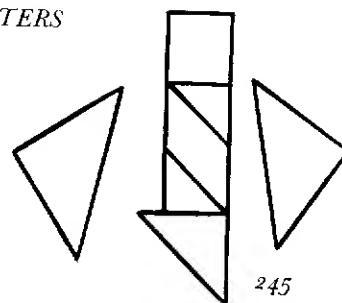
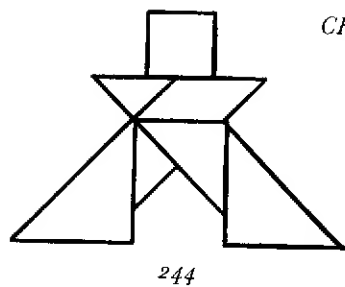


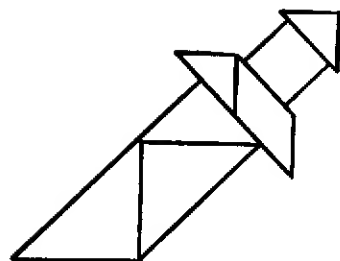
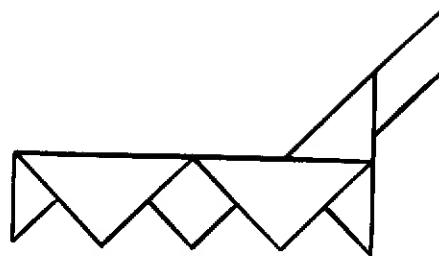
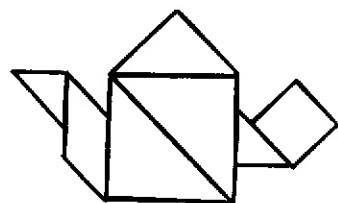
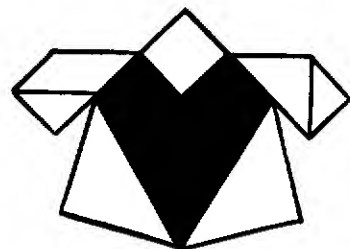
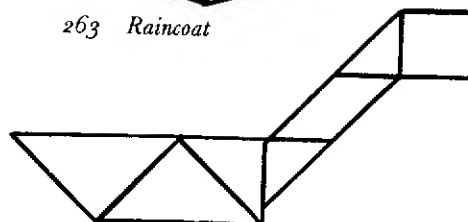
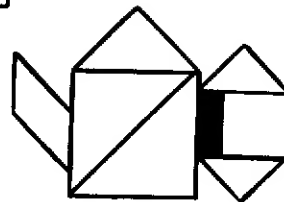
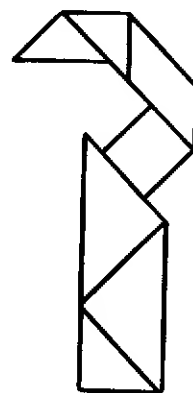
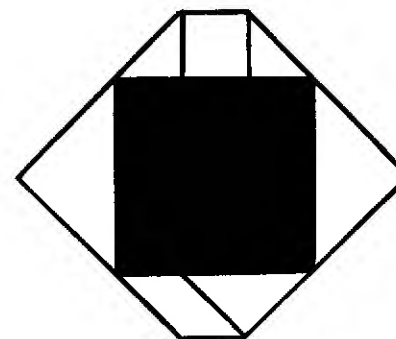
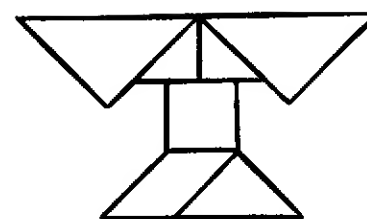
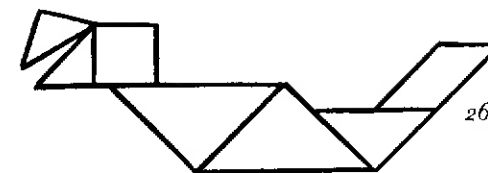
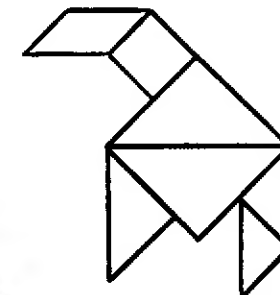
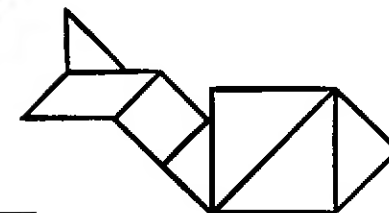
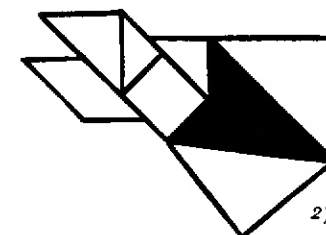
242

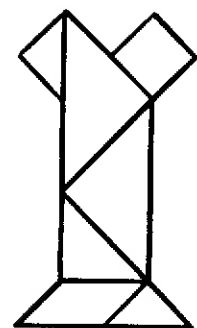


243

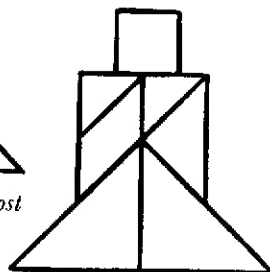
SOME
CHINESE
CHARACTERS



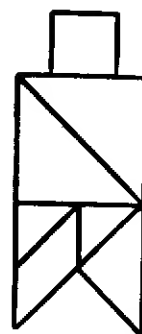
259 *Dagger*260 *Pinking knife*261 *Teapot*263 *Raincoat*264 *Soup spoon*265 *Teapot*262 *Mulberry hook*268 *Pendant*270 *Flower bowl*272 *Saw*266 *Dragon boat*267 *Three-legged toad*269 *Carp*271 *Scissors*



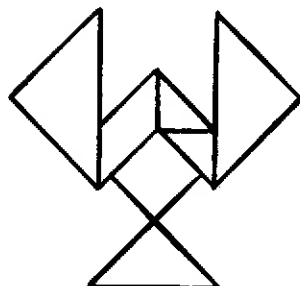
273 Clothes post



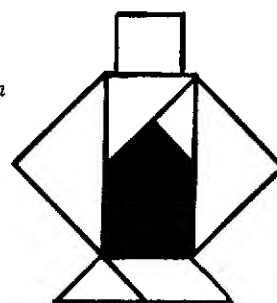
274 Bell



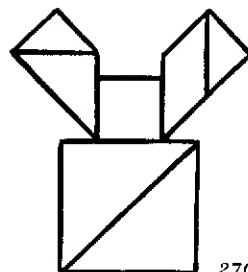
275 Feather tube



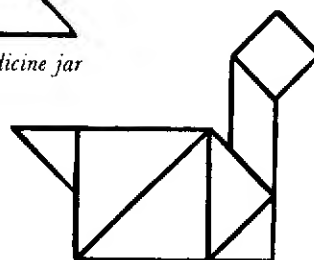
276 Lucky charm



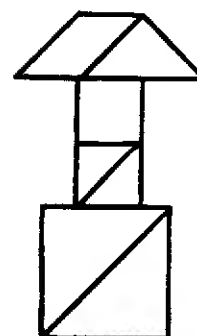
278 Medicine jar



279 Mirror rack

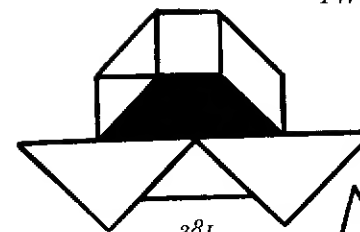


280 Ladle

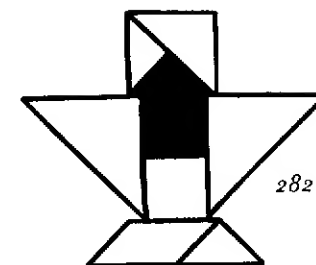


277 Look-out tower

TWO BASKETS



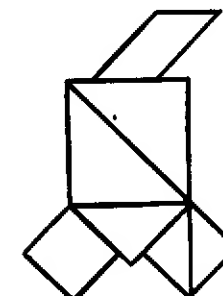
281



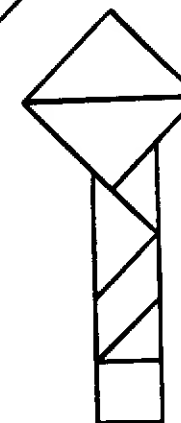
282

283
Arrow

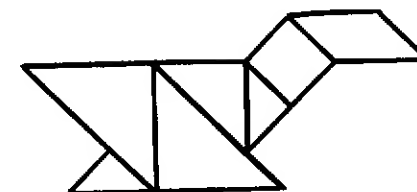
284 Candle



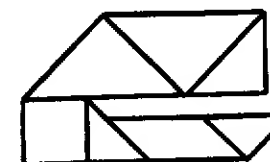
285 Water pot



287 Mallet



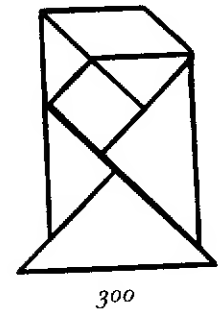
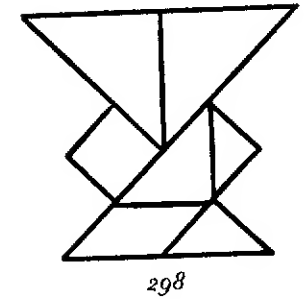
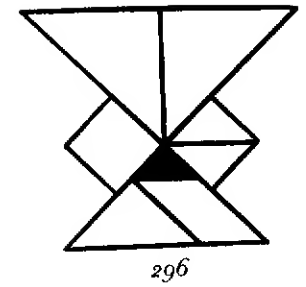
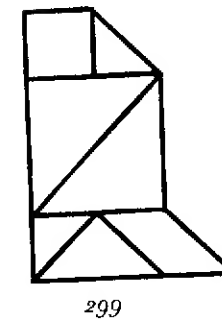
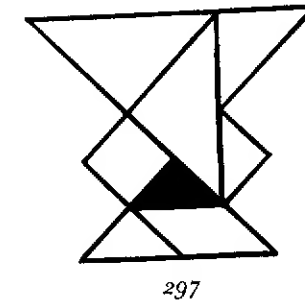
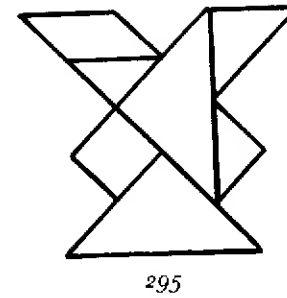
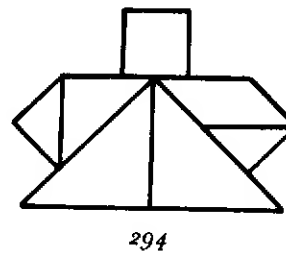
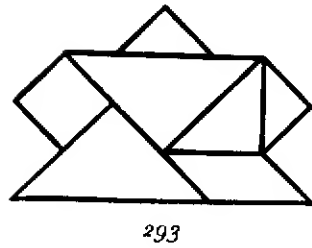
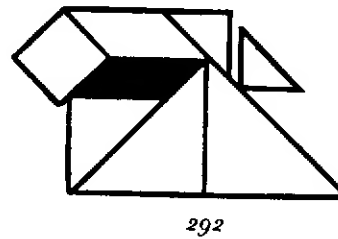
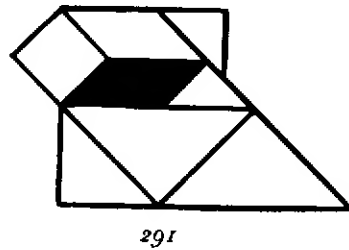
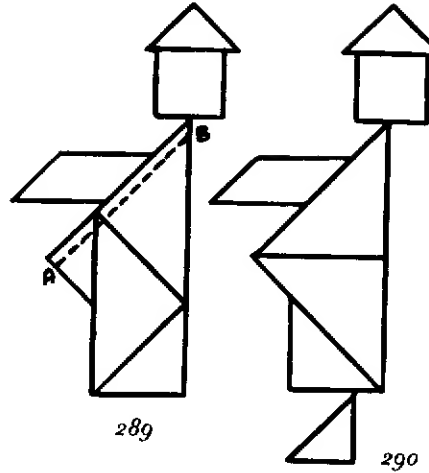
286 Measure



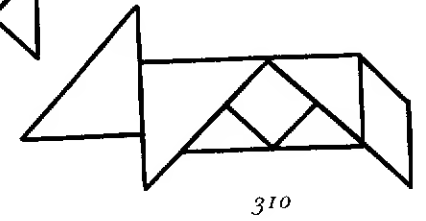
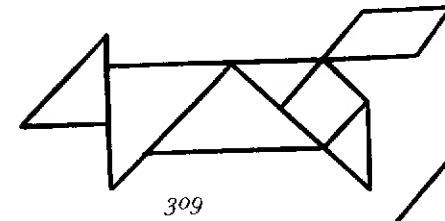
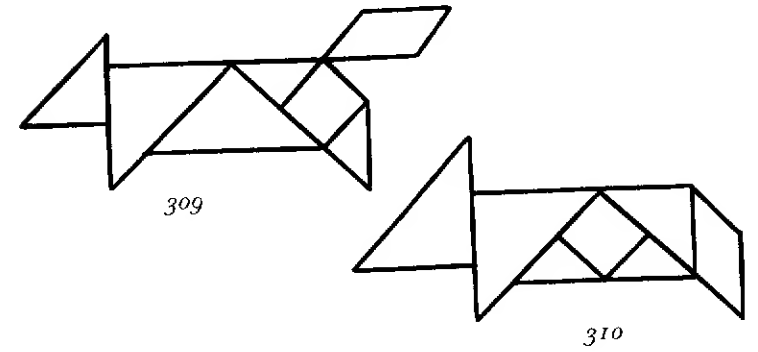
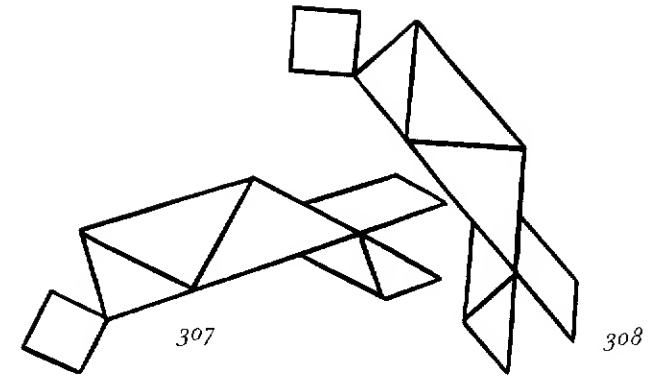
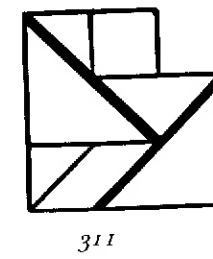
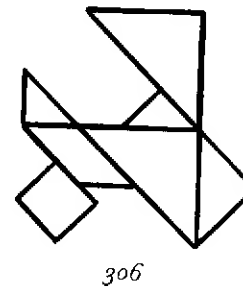
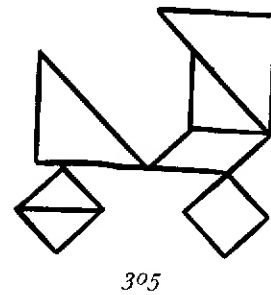
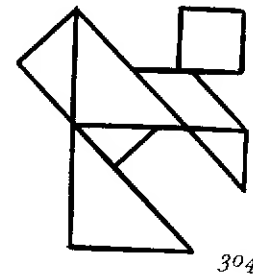
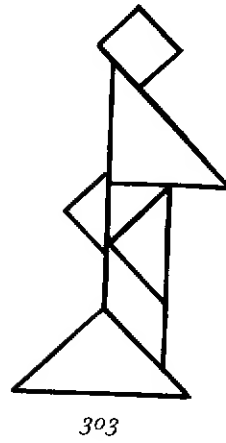
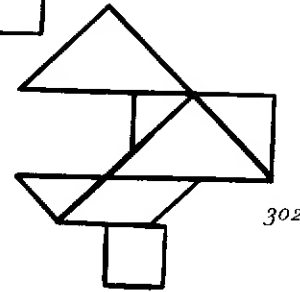
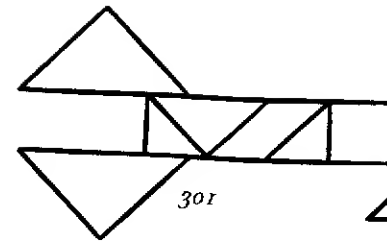
288 Penknife

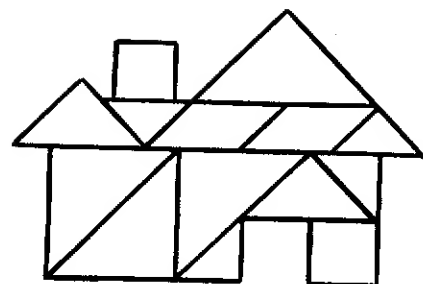
TANGRAM PARADOXES

For a resolution of the paradox in Tangrams 289 and 290 I cannot do better than repeat Dudeney's explanation: "It will be noticed that in both cases the head, hat and arm are precisely alike, and the width at the base of the body is the same. But this body contains four pieces in the first case, and in the second design only three. The first is larger than the second by exactly that narrow strip indicated by the dotted line between A and B. This strip is therefore exactly equal in area to the piece forming the foot in the other design, though when thus distributed along the side of the body the increased dimension is not easily apparent to the eye."

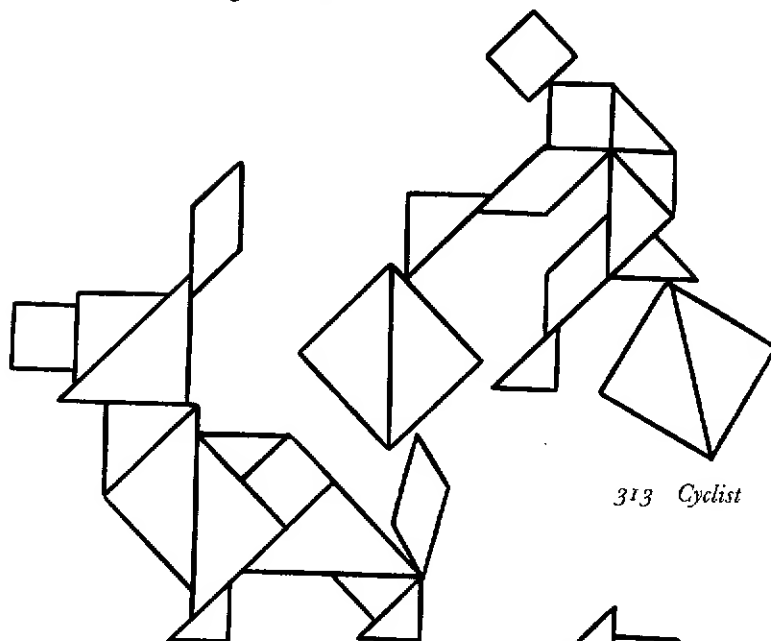


ILLUSIONS



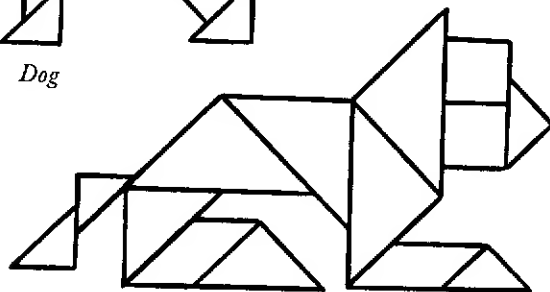


312 House

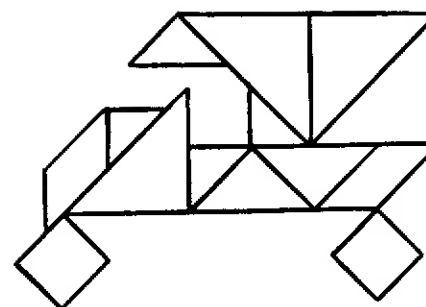


313 Cyclist

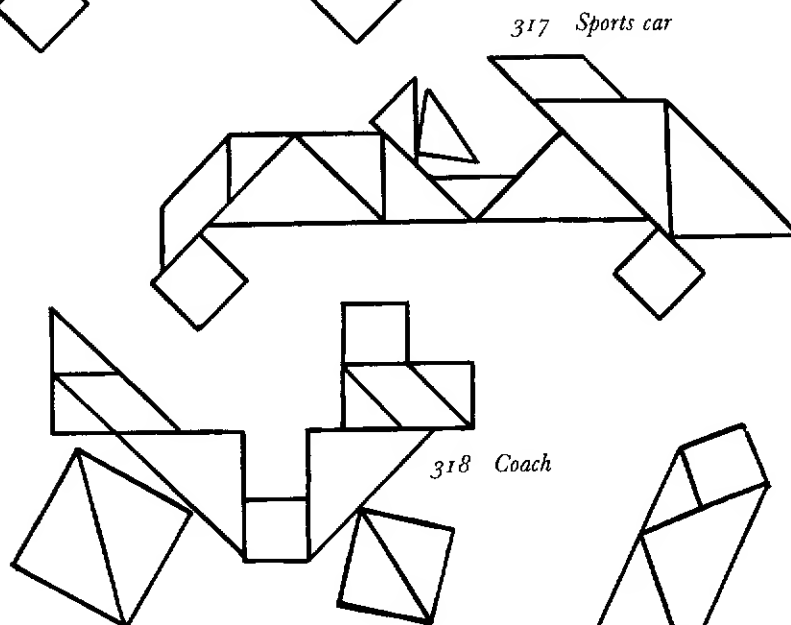
314 Dog



315 Lion

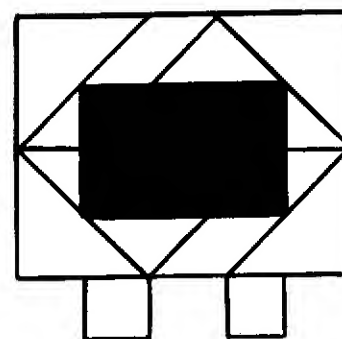


316 Car (ancient vintage)

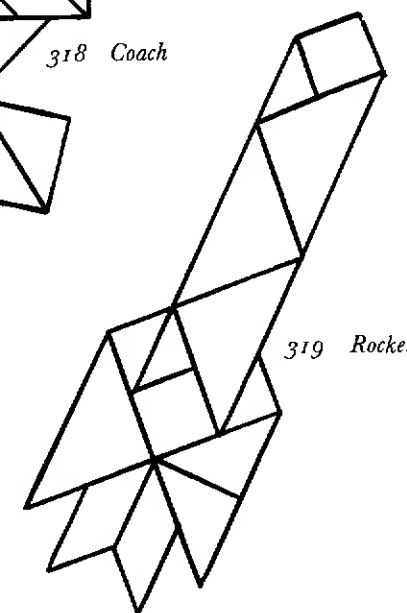


317 Sports car

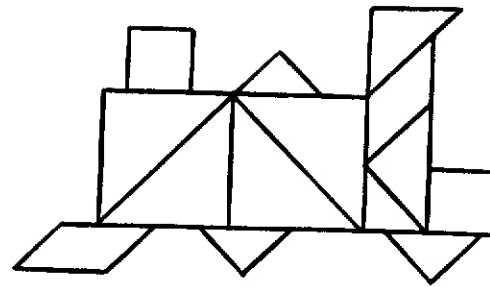
318 Coach



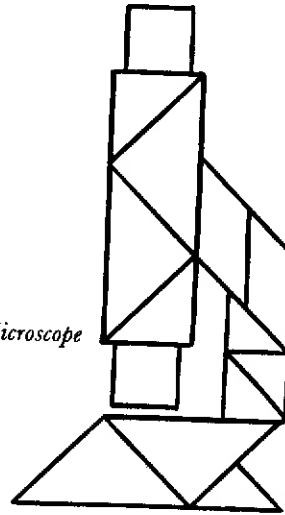
320 Television set



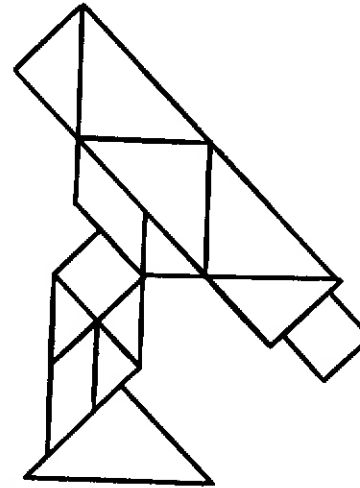
319 Rocket



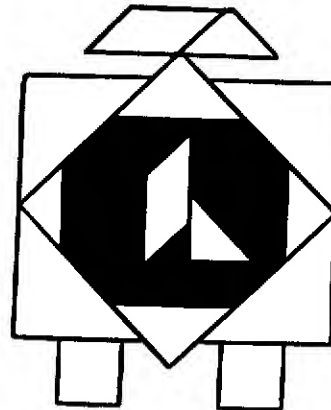
321 Locomotive



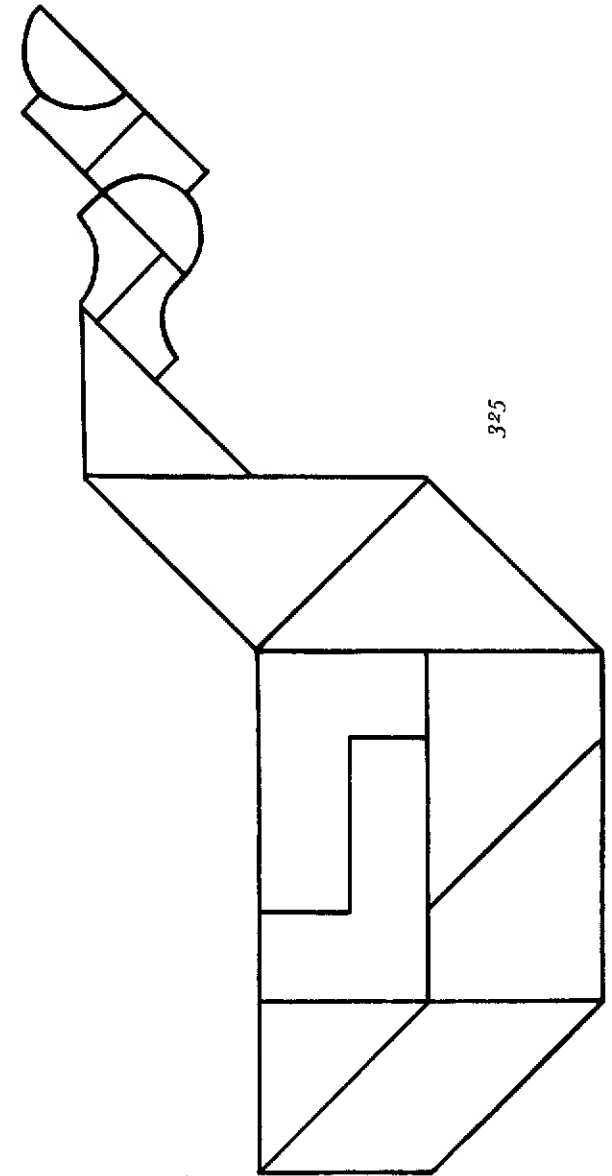
322 Microscope



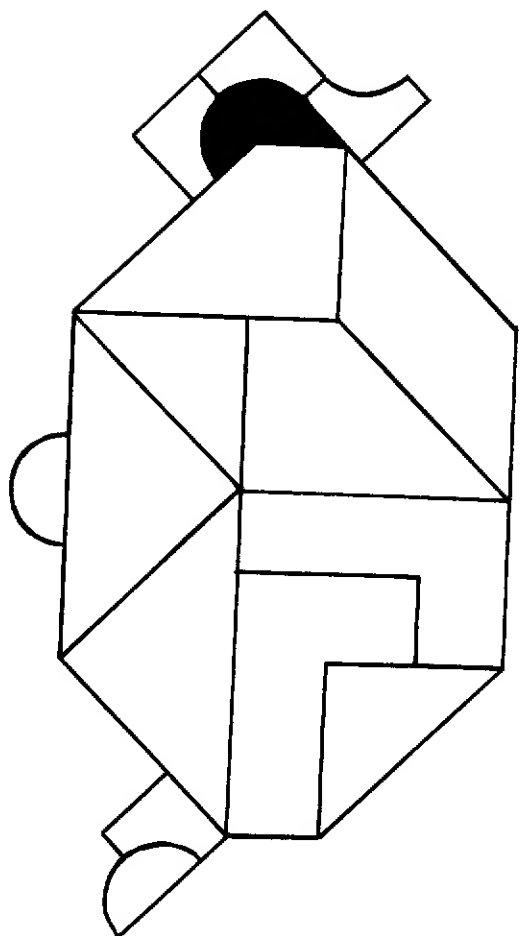
323 Telescope



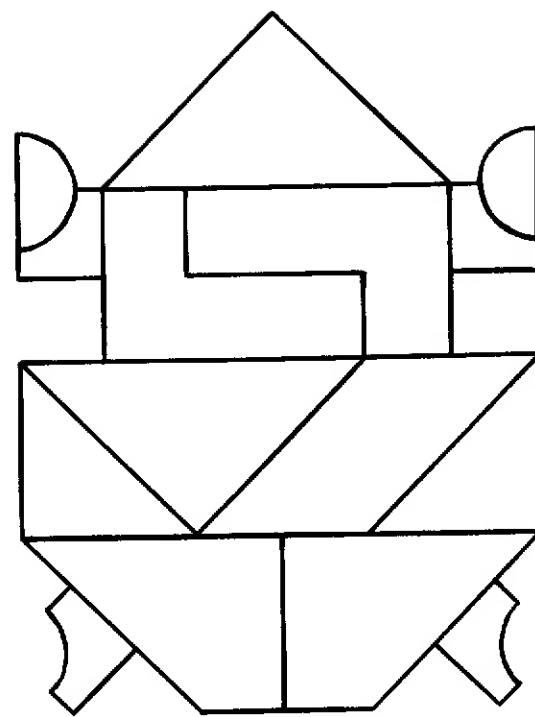
324 Alarm clock



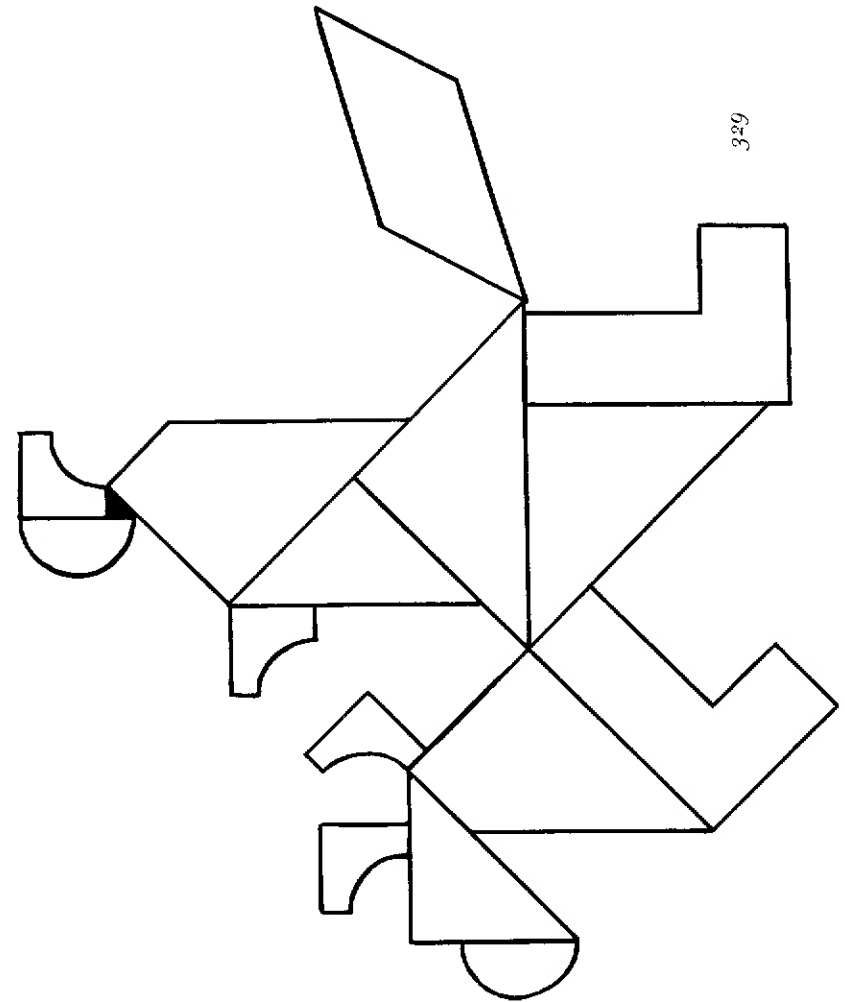
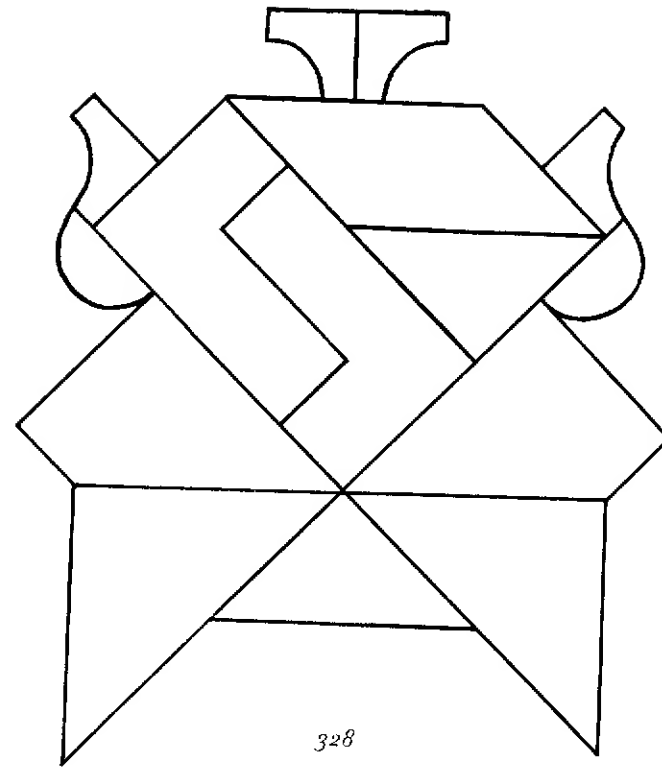
325

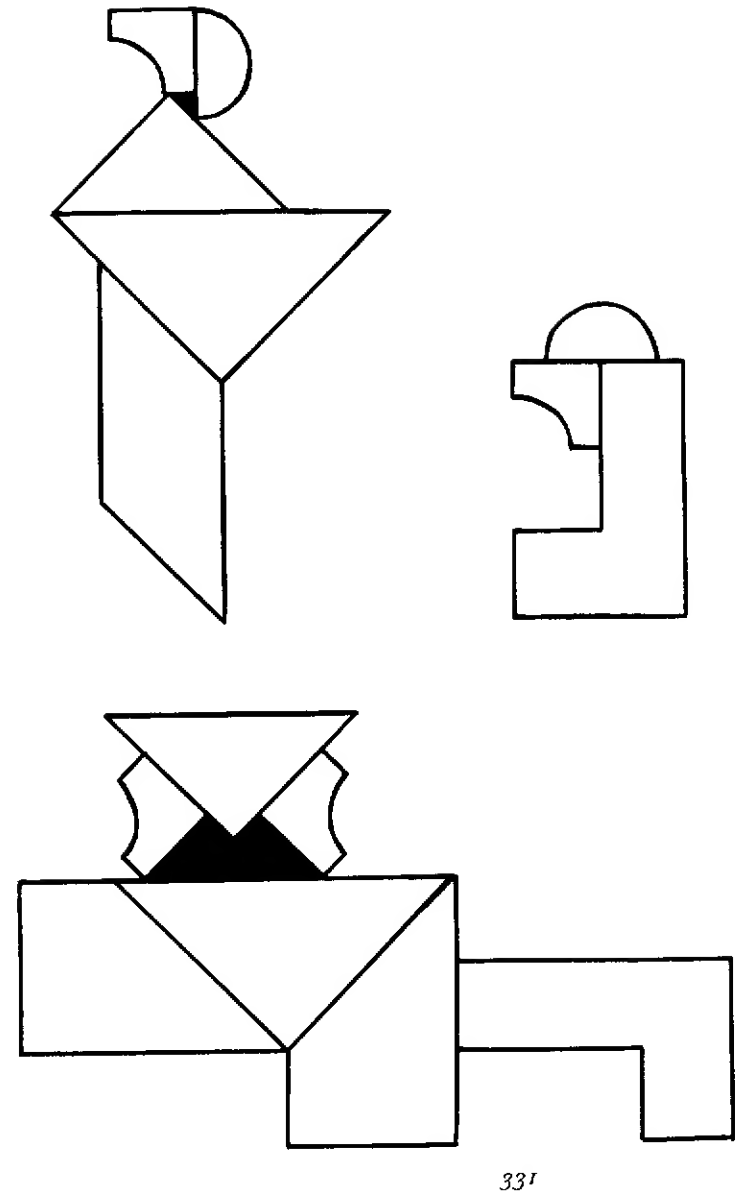
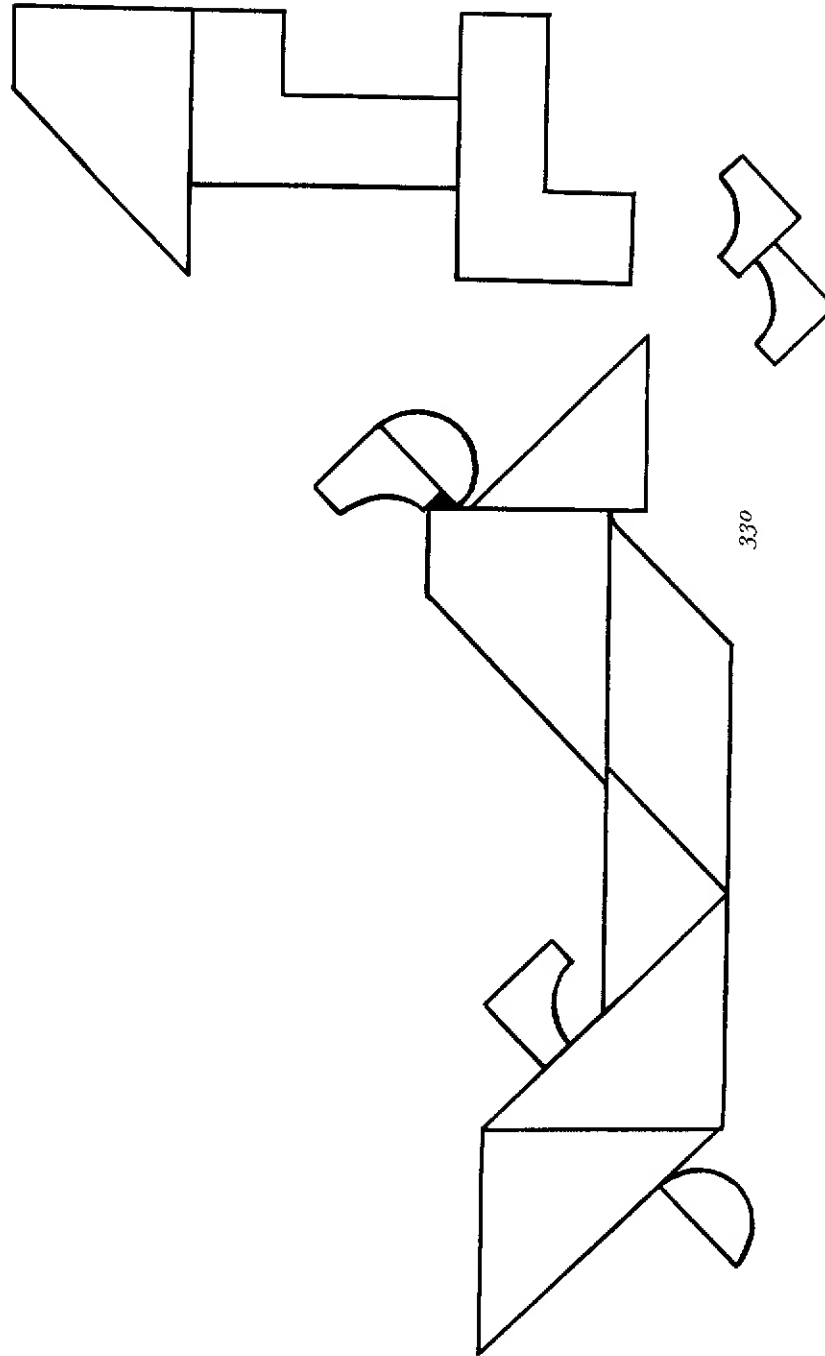


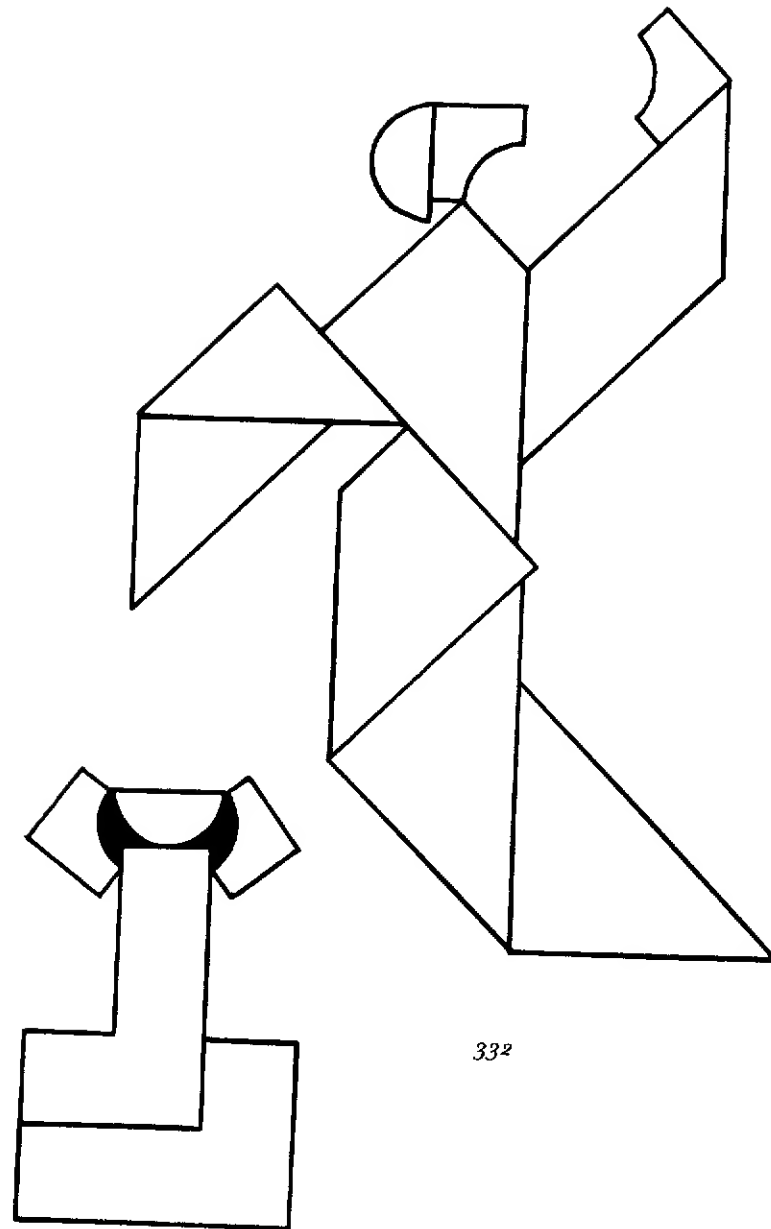
326



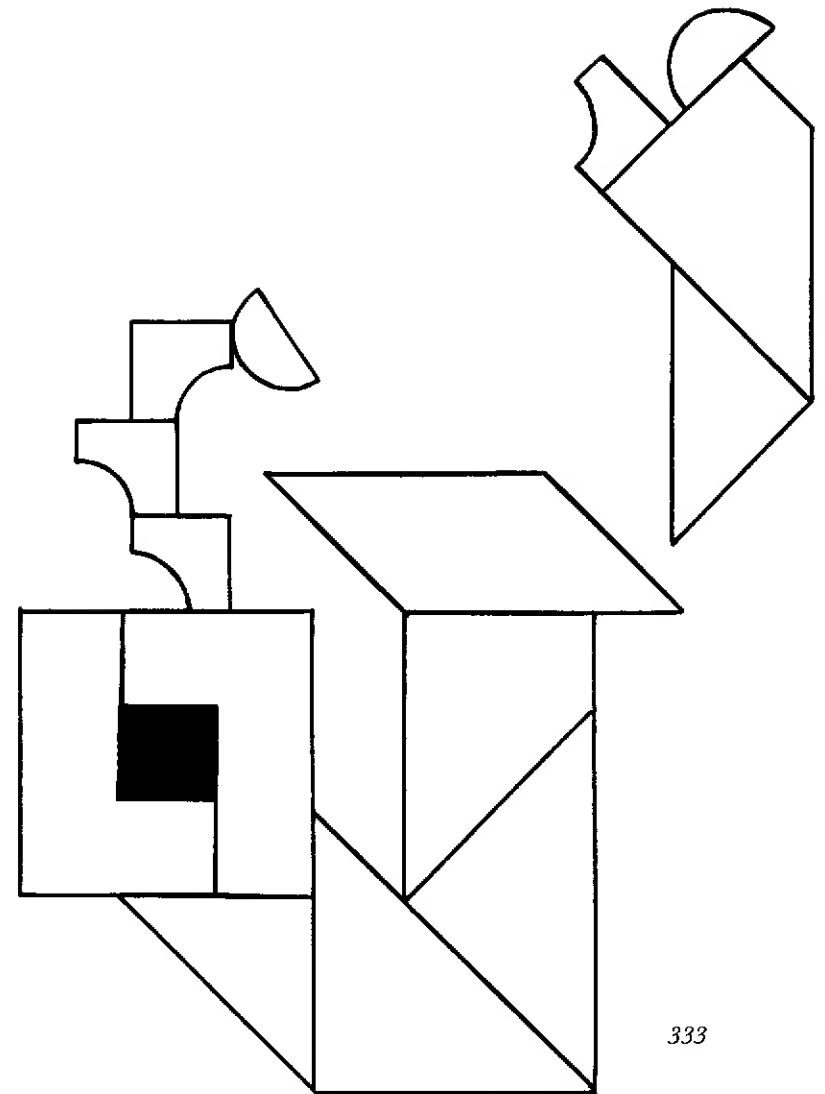
327





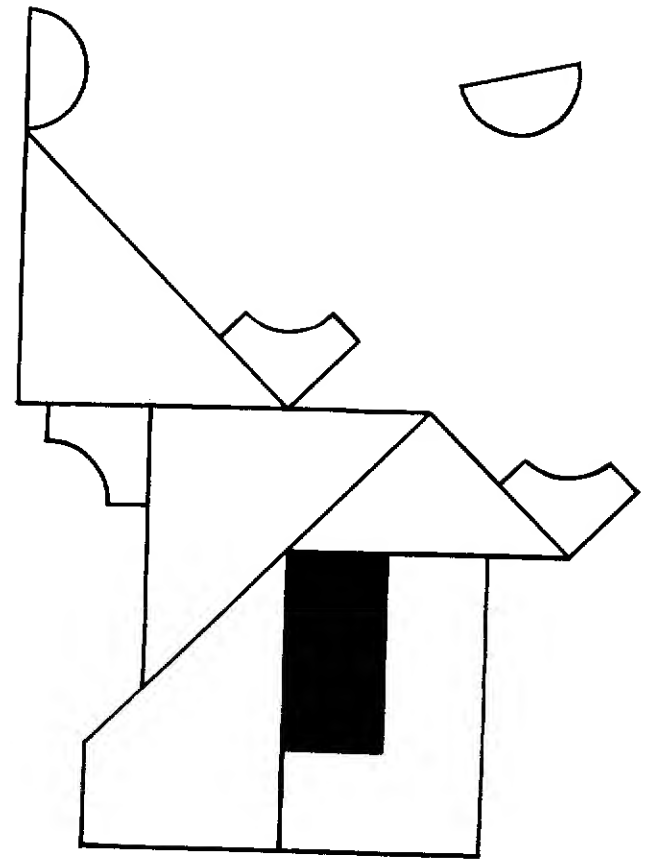


332



333

152



334

